Journal of Young Scientist, Volume X, 2023
ISSN 2344 - 1283; ISSN CD-ROM 2344 - 1291; ISSN Online 2344 - 1305; ISSN-L 2344 – 1283

CARL FRIEDRICH GAUSS – PRINCEPS MATHEMATICORUM

Alexandru-Valentin ANDREI, Gina BUJOR, Ioana MACOVEI, Andra VIȘAN

Scientific Coordinator: Lect. PhD Cosmin-Constantin NIȚU

Faculty of Land Reclamation and Environmental Engineering, University of Agronomic Sciences and Veterinary Medicine of Bucharest, 59 Mărăşti Blvd, District 1, 011464, Bucharest, Romania, Email: avandrei48@gmail.com, gina.bujor@gmail.com, ioanarobu80@gmail.com, andraa_16@yahoo.com

Corresponding author email: avandrei48@gmail.com

Figure 1. Carl Friedrich Gauss (1777-1855)

Abstract

Johann Friedrich Carl Gauss, was a German mathematician and physicist who made important contributions to many fields of mathematics and many different domains in science. Sometimes referred to as the Princeps mathematicorum (Latin for "the foremost of mathematicians") and "the greatest mathematician since antiquity", Gauss is considered among the most influential mathematicians throughout history.

Keywords*: Carl Friedrich Gauss, geodesy, mathematics, mathematical work, science.*

INTRODUCTION

Carl Friedrich Gauss, original name Johann Friedrich Carl Gauss, was a German mathematician and a child prodigy. He was able to calculate before he could even talk and his contributions to number theory, the theory of functions, probability theory, geodesy, geometry, planetary astronomy and potential theory made him one of history's greatest and most influential mathematicians.

EARLY YEARS

Carl Friedrich Gauss was born on April 30, 1777, in Brunswick, Germany. His parents were Gebhard Dietrich Gauss, a gardener and bricklayer, and Dorothea Benze, the daughter of a well-to-do family. From an early age, Gauss showed exceptional mathematical abilities, and he quickly became fascinated with numbers and problem-solving. His mother was illiterate and never recorded the date of his birth, the only clue about his birthday was that he born on a Wednesday, 39 days after Easter. Later, Gauss discovered the mystery of his birthdate with methods to compute the date of Easter in both past and future years.

As a child, Gauss was mostly self-taught, as his parents were not educated enough to provide him with formal instruction. Nevertheless, he made remarkable progress in mathematics and other subjects, and he began to attract the attention of local scholars and patrons.

ELEMENTARY SCHOOL

At the age of 7, in the year 1784, he began attending a public elementary school. An often-told story, with varying details, is that his primary school teacher, J.G. Büttner, attempted to occupy his students by having them add up the integers from 1 to 100. However, to the surprise of all, the young Gauss arrived at the correct answer almost instantaneously, thanks to a sudden mathematical insight. Gauss had realized that adding up pairs of terms from opposite ends of the list would result in identical intermediate sums.

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2} \qquad (1)$$

At the age of ten, he became friends with a teacher's assistant who helped him obtain mathematics books that they studied together.

HIGH SCHOOL AND UNIVERSITY

After completing elementary school, he was admitted to a gymnasium (high school) in 1788. Within just two years, he had excelled to such a remarkable degree that he was presented to the Duke. As a reward for his academic skills, Gauss was given a fellowship to attend the Collegium Carolinum, which is now known as the Technische Universität Braunschweig. He studied there from 1792 to 1795 before moving on to the University of Göttingen, where he continued his studies from 1795 to 1798.

At the age of 19, Gauss achieved a significant breakthrough in mathematics by demonstrating that a straightedge and a compass could construct any polygon with a number of sides equal to a Fermat prime. This proof marked the first advancement in polygon construction in over two millennia. The construction of a regular heptadecagon, a 17-sided polygon, using this method while he was still a student, was a groundbreaking discovery that inspired Gauss to pursue a career in mathematics.

In 1796, Gauss accomplished another remarkable step by becoming the first mathematician in the world to prove the law of quadratic reciprocity in number theory, which he named the "fundamental/golden theorem." That same year, he also formulated the prime number theorem, although he did not publish it.

Gauss usually refused to present the intuition behind his often highly sophisticated evidence - that they appeared "suddenly" and erased any trace of how he discovered them, which is justified, although overwhelming in the work “Disquisitiones Arithmeticae”, published when Gauss was 24 years old, in 1801. This seminal work served as the basis for the systematic study of number theory as a distinct field of inquiry. During his studies at the University of Göttingen, Gauss made a notable discovery, which he detailed in Section VII of his book. Additionally, he introduced the symbol for congruence in geometry.

DISSERTATION

In 1799, as part of his doctoral dissertation, Gauss presented a proof of the fundamental theorem of algebra, which asserts that every polynomial equation with complex coefficients has at least one complex root. This work represented an important contribution to the field of mathematics. Additionally, Gauss made significant strides in determining the number of solutions for polynomial equations with coefficients in finite fields. His findings formed the foundation for the Weil conjectures in 1949.

In the field of differential geometry, Carl Gauss introduced the concept of "Gaussian curvature" as a measure of intrinsic curvature. This concept led to the development of Gauss's Theorema Egregium, a fundamental result in differential geometry. Gauss's work in this area remains influential to this day.

APPRAISAL OF GAUSS

The British mathematician Henry John Stephen Smith (1826–1883) gave the following appraisal of Gauss: “If we except the great name of Newton it is probable that no mathematicians of any age or country have ever surpassed Gauss in the combination of an abundant fertility of invention with an absolute rigorousness in demonstration, which the ancient Greeks themselves might have envied. It may seem paradoxical, but it is probably true that it is precisely the efforts after logical perfection of form which has rendered the writings of Gauss open to the charge of obscurity and unnecessary difficulty. Gauss says more than once that, for brevity, he gives only the synthesis, and suppresses the analysis of his propositions.” (Smith H.J.S.)

RELIGIOUS VIEWS

Gauss was a Protestant Lutheran, a member of st. John the Evangelist Lutheran Church. Albani of Göttingen.One of his biographers, G. Waldo Dunnington, described Gauss' religious conceptions in these terms: “For him, science was the means of discovering the immortal core of the human soul. It entertained him on days when he was plentiful and, through the views he opened, offered him comfort. At the end of his life, that gave him confidence. The Gaussian God is not a cold and distant invention of metaphysics or a twisted caricature of bittersweet theology. No one can guarantee the completeness of knowledge enough to justify his arrogant assertion that clairvoyance is all light, and no one else can report the truth like him. For Gauss, it was not the man who whispered his creed, but the man who put it into practice. He believed that being worthy on earth was the best and only preparation for heaven. Religion is not a matter of literature, it is a matter of life. Divine revelation is continuous and is not contained in sacred tablets or scrolls.” (Dunnington, 1955).

PERSONALITY

Karl Gauss was a passionate perfectionist and a hard worker. He was never a prolific writer and refused to publish works that he considered incomplete and uncritical. This matched his personal motto “pauca sed mature” ("small but ripe"). His personal diaries show that he made some important mathematical discoveries years and decades before they were published by his contemporaries. Math historian Eric Temple Bell said that if Gauss had published all his findings in time, he would have made 50 years of progress in mathematics. (Bell E.T., 2009)

Despite his introverted personality trait, Gauss was highly respected by his colleagues and contemporaries for his contributions to the fields of mathematics, physics, and astronomy. He was known for his meticulous attention to detail and his ability to solve complex problems with great accuracy and speed.

Gauss was also a deeply patriotic individual who was proud of his German heritage and identity. He was known for his support of the German language and culture, and played a role in the development of a standard system of measurement for the German states.

In addition to his intellectual pursuits, Gauss was also interested in music. He played the piano and organ. He was married twice and had six children, two of whom also became mathematicians.

LATER YEARS AND DEATH

In 1831, Gauss collaborated successfully with the physicist Professor Wilhelm Weber, introducing new knowledge of magnetism and discovering Kirchhoff's laws of electrical circuits. During this time, he developed the law of the same name. They built the first electromechanical telegraph in 1833, linking the observatory to the Institute of Physics in Göttingen. Gauss ordered the construction of a magnetic observatory in the Observatory Garden and together with Weber founded the "Magnetischer Verein", which allows the

measurement of the Earth's magnetic field in many regions of the world.
Gauss died on 23 February 1855 in Göttingen (then the Kingdom of Hanover and now Lower Saxony) ("Carl Friedrich Gauss". Wichita State University.) and he was buried there in Albany Cemetery. Two people commended his funeral: Gauss's son-in-law Heinrich Ewald and Gauss's close friend and biographer, Wolfgang Sedorius von Waltershausen (Wolfgang Sartorius von Waltershausen). Rudolf Wagner retained and studied his brain and found that it weighed 1,492 grams (slightly above average) and had a brain area of 219,588 mm2. (This reference from 1891 (Donaldson H., 1891) says: "Gauss, 1492 grm. 957 grm. 219588. sq. mm."; i.e. the unit is mm2. In the later reference: Dunnington (1927), the unit is erroneously reported as square cm, which gives an unreasonably large area; the 1891 reference is more reliable.) (340,362 square inches).
He also discovered highly developed circumvolutions, which were believed to explain his genius at the beginning of the twentieth century (Bardi, 2008). Throughout his lifetime, his discoveries were recorded in his personal journal, but he did not publish them. These notable findings include the method for fitting least squares, the Cauchy integral theorem for analytic functions, and non-Euclidean geometry.

AWARDS AND ACHIEVEMENTS

In 1810, he was honored with the Lalande Prize by the French Academy of Sciences for recognizing his contributions to astronomy.
He received the prize of the Danish Academy of Sciences in 1823 for the study of maps that preserve the angle.
He was presented with the Copley Medal by the Royal Society, London, in 1838 "for his inventions and mathematical research in magnetism".

THE GAUSS PRIZE

The Gauss Prize is to honor scientists whose mathematical research has had an impact outside mathematics – either in technology, in business, or simply in people's everyday lives.The prize is awarded jointly by the Deutsche Mathematiker-Vereinigung (German Mathematical Union) and the International Mathematical Union.

Material - Au 585/00 (14KT Gold)
Diameter - 65 mm
Weight - 155.5 g
Cutting depth - 0,25-0,30 mm, embossed outer edge
Package - Case of mahogany
The 2010 cash value of the Gauss Prize 10,000 EUR.

Figure 2. The Gauss Medal

Interpretation of the medal

Soon after Giuseppe Piazzi discovered the celestial body Ceres on January 1st, 1801, Ceres disappeared from view, and there were no reliable techniques available to predict its orbit from Piazzi's limited observational data. Introducing a revolutionary new idea, the now well-known least squares method, Carl Friedrich Gauss was able to calculate Ceres' orbit in a very precise way, and in December 1801 Ceres was rediscovered by the astronomer Zack very close to the predicted position.

This impressive example illustrating the power of the applications of mathematics provided the general idea for the design of this medal.

Dissolved into a linear pattern, the Gauss effigy is incomplete. It is the viewer's eye which completes the barcode of lines and transforms it into the portrait of Gauss.

A similar pattern, accomplished by horizontal lines, is one of the features on the back of the medal. This grid is crossed by a curve. The disk and the square, two elements connected by the curve, symbolize both the least squares method and the discovery of Ceres' orbit.

The mathematical language has been reduced to its most fundamental elements, such as point, line and curve. Moreover, these elements represent natural processes. The imagery of the medal is a synthesis of nature's and mathematics' sign language.

Medal designer Jan Arnold.

MATHEMATICAL WORK

Gauss published more than 150 works which had a great impact on different scientific domains. We mention below some of his contributions.

1. Construction of Heptadecagon

Gauss constructed a regular 17-sided polygon, heptadecagon with the help of straightedge and compass only, when he was just 19. His proof says that the number of sides of the regular polygon is distinct Fermat primes, which are of $F_n = 2^{2^n} + 1, \quad n \in \mathbb{N}$. Gauss discovered a general formula for construction polygon beyond heptadecagon. He further constructed other n-sided figures. Gauss-Wantzelț's theorem (completely proven in 1873) states that a regular n-sided polygon can be constructed with the straightedge and the compass if and only if

$$n = 2^k F_1 F_2 \ldots F_m \qquad (2)$$

where $F_1 F_2 \ldots F_m$ are Fermat primes.

Gauss was so happy with his result that he asked the stones man to inscribe a heptadecagon over his tomb. However, the stoneman declined by saying that the difficult construction of a heptadecagon will look like a circle only.

2. Integers as sum of Triangular Numbers

In 1796, Gauss discovered that every positive integer can be expressed as the sum of at most three triangular numbers, i.e. numbers of the form $\Delta = \frac{n(n+1)}{2}$.

He wrote this result in his diary as,

“ΕΥΡΗΚΑ num = Δ + Δ + Δ“ (3)

This diary was lost after 40 years of his death. Gauss was supposed to be the last person who mastered every aspect of mathematics.

3. Complex Numbers

Even though complex numbers are known from the 16th century, Gauss was the first mathematician to give a clear picture of complex numbers and functions of complex variables. Although Euler worked on imaginary and complex numbers, there was still no proper explanation of how real and imaginary numbers are related. Gauss took into practice the complex numbers and gave the standard notation a+ib for complex numbers. Then onwards, more concepts of complex numbers were unleashed.

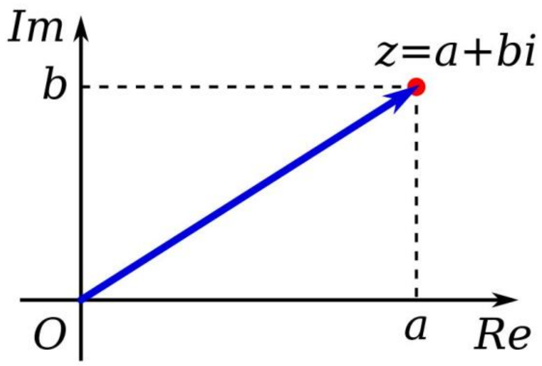


Figure 3. Complex numbers

4. Fundamental Theorem of Algebra

When he was just 22, Gauss gave the proof of the Fundamental Theorem of Algebra, which states that every non-constant single-variable polynomial over the complex numbers has at least one root.

He also proved that the field of complex numbers $\mathbb{C}$ is algebraically closed (a polynomial with complex coefficients has at least one complex root) unlike the field of real numbers $\mathbb{R}$, where a polynomial with real coefficients may have only complex roots.

5. Contributions to Number Theory

Gauss used to say that “Mathematics is the queen of sciences and number theory is the queen of mathematics." In 1801, at the age of 24, Gauss published his most famous book "Disquisitiones Arithmeticae", written in latin, which is considered the most influential book in the field of number theory. This book includes,

- Triple bar symbol (≡) for congruence;
- Presentation of Gauss' method of modular arithmetic;
- The first two proofs of the law of quadratic reciprocity;
- Theories of binary and ternary quadratic forms;
- Gauss class number problem;
- Proved Fermat's polygonal number theorem for n =5;
- Descartes' rule of signs
- Kepler's conjecture for regular arrangements
- Calculating the date of Easter
- Discovered Cooley-Tukey FFT algorithm for calculating Discrete Fourier transforms (rediscovered and popularized);

6. Properties of Pentagramma Mirificum

Some properties of Pentagramma mirificum were discussed by Gauss. Pentragramma mirificum is a star-shaped polygon on a sphere whose sides are great circle arcs and all the interior angles are right angles.

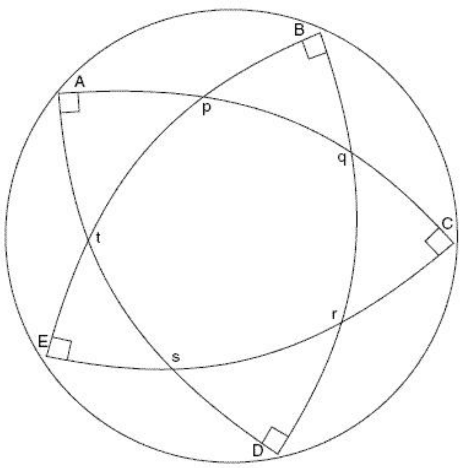


Figure 4. Pentagramma Mirificum

Gauss introduced the following expression

$$(\alpha, \beta, \gamma, \delta, \varepsilon) = (\tan^2 TP, \tan^2 PQ, \tan^2 QR, \tan^2 RS, \tan^2 ST) \quad (4)$$

, where the following identities hold and three quantities can be determined by the remaining two.

$$1 + \alpha = \gamma\delta, 1 + \beta = \delta\varepsilon, 1 + \gamma = \alpha\varepsilon, 1 + \delta = \alpha\beta, 1 + \varepsilon = \beta\gamma \quad (5)$$

Gauss gave a solution as (α, β, γ, δ, ε) = (9, 2⁄3, 2, 5, 1⁄3) and also proved the following identity:

$$\alpha\beta\gamma\delta\varepsilon = 3 + \alpha + \beta + \gamma + \delta + \varepsilon = \sqrt{(1+\alpha)(1+\beta)(1+\gamma)(1+\delta)(1+\varepsilon)} \quad (6)$$

, and area of polygon PQRST is given by,

APQRSTAPQRST = 2π – (|PQ| + |QR| + |RS| + |ST| + |TP|). (7)

7. Gaussian Function and The Gaussian Error Curve

Gauss introduced Gaussian (normal) distribution and demonstrated how probability can be represented graphically by a bell-shaped or a normal curve. This curve has the highest peak around mean and expected value and gradually falls off near plus and minus infinity. The graph of the normal curve is given below.

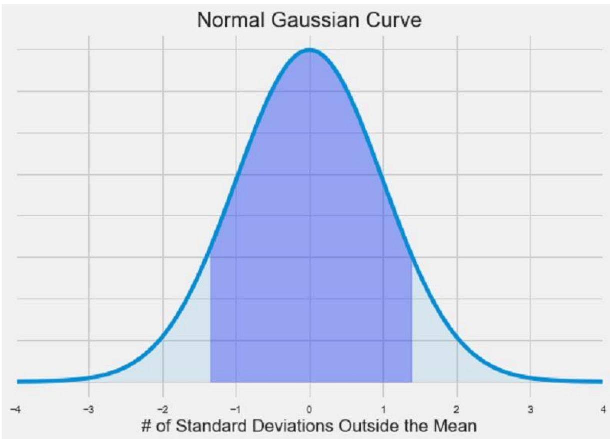


Figure 5. Normal Gaussian Curve

Also, Gaussian distribution is a type of continuous probability distribution for a real-valued random variable and the general form of the density function is given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{2}\right)^2} \quad (8)$$

8. Theorema Egregium ("Remarkable Theorem")

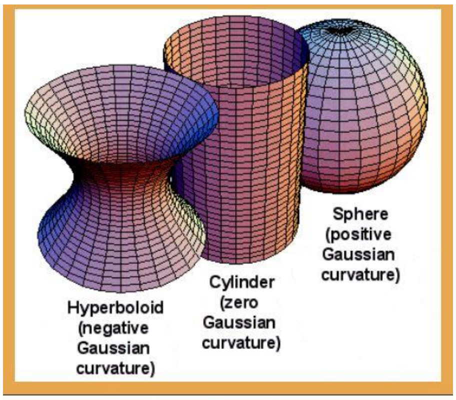
Gauss' Theorema Egregium is a result of differential geometry that talks about the curvature of surfaces. The theorem states that,

The Gaussian curvature of a surface does not change if one bends the surface without stretching it."

The Gaussian curvature K of a smooth surface in three-dimensional space at a point is the product of the principal curvatures, $k_1$and k_2, at the given point:

$$K = k_1 k_2 \qquad (9)$$

Thus, the theorem means that the Gaussian curvature can be determined entirely by measuring angles, distances and their rates on a surface, without reference to the particular manner in which the surface is placed, in the ambient 3-dimensional Euclidean space. The theorem is said to be remarkable because the result does not depend on its embedding even after undergoing all bending and twisting deformations.



Gaussian curvature

Figure 6. Gaussian curvature

9. Gauss-Ostrodradski's Theorem

Gauss theorem is also known as the Divergence theorem or Ostrogradsky's theorem. In vector calculus, this theorem states that, the surface integral of a vector field over a closed surface, which is called the flux through the surface, is equal to the volume integral of the divergence over the region inside the surface."

This theorem establishes a relationship between the flux of a vector field over a closed surface and the volume integral of the divergence of the field. Graphical representation of the Gauss theorem is given by,

$$\iiint_V (\nabla \cdot f)dv = \oiint_S (F \cdot n)ds \qquad (10)$$

10. Gauss' irreducibility Lemma

Gauss's lemma states that the product of two primitive polynomials is primitive (a polynomial with integer coefficients is primitive if it has 1 as the greatest common divisor of its coefficients). This lemma over the integers is stated as:

"If P(X) and Q(X) are primitive polynomials over the integers, then product P(X)Q(X) is also primitive."

Gauss irreducibility lemma states that:

"A non-constant polynomial in $\mathbb{Z}[X]$ is irreducible in $\mathbb{Z}[X]$ if and only if it is both irreducible in $\mathbb{Q}[X]$ and primitive in $\mathbb{Z}[X]$."

11. Gauss Approximation Method

Gauss had given Gauss-Siedel iterative method in numerical algebra used to solve a system of linear equations. This method is named after German mathematicians Gauss and Phillip Ludwig von Siedel. This method is used to solve a system of n linear equations with unknown X

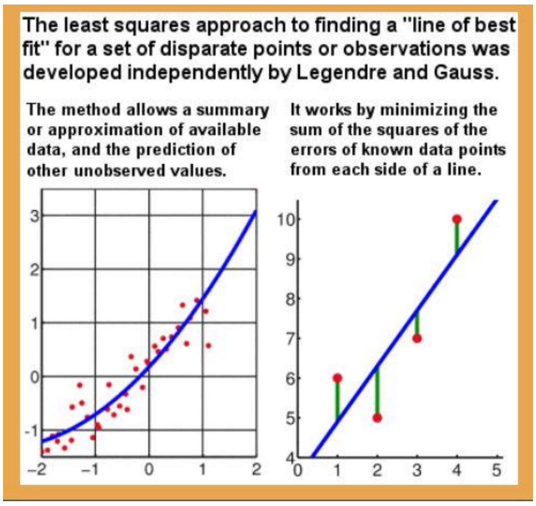
$$AX = b \qquad (11)$$

and the iteration is given by:

$$LX_{k+1} = b - UX_k, \qquad (12)$$

where X_k is the k-th approximation of X. A is decomposed into a lower triangular matrix L, and a strictly upper triangular matrix U, i.e., A = L + U.

12. THE LEAST SQUARES METHOD



Line of best fit by Gauss' least squares method

Figure 7.

13. GEODETIC SURVEY

In 1818 Gauss, putting his calculation skills to practical use, carried out a geodetic survey of the Kingdom of Hanover, linking up with previous Danish surveys. To aid the survey, Gauss invented the heliotrope, an instrument that uses a mirror to reflect sunlight over great distances, to measure positions.

In 1828, when studying differences in latitude, Gauss first defined a physical approximation of the figure of the Earth as the surface everywhere perpendicular to the direction of gravity (of which mean sea level makes up a part); later his doctoral student Johann Benedict Listing called this the geoid.

REFERENCES

Bardi J., (2008). The Fifth Postulate: How Unraveling A Two Thousand Year Old Mystery Unraveled the Universe. John Wiley & Sons, Inc. p. 189. ISBN 9780470467367.

Bell E.T., "Ch. 14: The Prince of Mathematicians: Gauss". Men of Mathematics: The Lives and Achievements of the Great Mathematicians from Zeno to Poincaré. New York: Simon and Schuster, pp. 218–269 ISBN 0671464000.

Donaldson H., "Anatomical Observations on the Brain and Several Sense Organs of the Blind Deaf Mute, Laura Dewey Bridgman ". The American Journal of Psychology. E. C. Sanford. 4 (2): 248–294. doi:10.2307/1411270. JSTOR 1411270.

Dunnington G.W., Carl Friedrich Gauss, Titan of Science: A Study of His Life and Work. Exposition Press, pp. 298301.

Smith H.J.S, Presidential Address, Proceedings of the London Math. Soc. VIII, 18

https://studiousguy.com/carl-friedrich-gauss-contributions-in-mathematics/

https://www.storyofmathematics.com/19th_gauss.html/

https://www.mathunion.org/imu-awards/carl-friedrich-gauss-prize

Carl Friedrich Gauss - Wikipedia