

ANALYSIS OF PLANIMETRIC ACCURACY OF OLD MAPS OF TRANSYLVANIA

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Abstract

The target of this paper is the analysis of some old maps of Transylvania from different periods of time in order to obtain information about the planimetric position accuracy of cities and also information about the maps. In the first part will be presented each map and operations used to analyse those maps. In the second part the results will be presented and compared in order to get to a conclusion.

Key words: accuracy, Helmert transformation, Mercator projection, old maps.

INTRODUCTION

In this paper the main objective is to determine the accuracy of some old maps of Transylvania designed in period of time of almost three centuries. Our research starts with the map of Transylvania that has a mathematical support, map made by Gerardus Mercator in 1616, and ends with a map published in 1862 made in a surveying campaign during the period of Austro-Hungarian domination. Knowing the accuracy of the old maps we can determine how much we can trust in those maps in order to obtain historical and geographical information in the future researches.

MATERIALS AND METHODS

We have analysed three maps: first one is from 1616 made by Gerardus Mercator, the second is made by Johann Schreiber in 1730 and the last one in 1862 by an unknown author.

The first map studied is "Transsylvania" (figure 1), made by the great geographer, cartographer and mathematician Gerardus Mercator. We assumed that this map was realised in the projection that bears his name. This map is the first map of Transylvania that has a network of meridians and parallels, being the first with a

mathematical support. Previous maps of this region are considered to be only paintings or drawings realised with empirical methods.

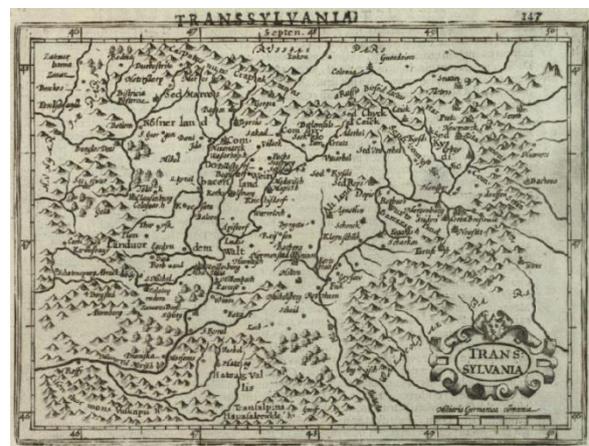


Figure 1. „Transsylvania”, Gerardus Mercator, 1616

As we can observe on this map, the longitude is translated to East with about 23 degrees. The position of the details like relief, hydrographic network and human settlements indicates also that the map is rotated with an angle that differs from the actual maps orientation.

This map has in the bottom right corner a graphic scale divided in German miles. Knowing that a German mile was almost 8 kilometers in that period we can assume that the scale of this map is about 1:2 000 000.

The names of the settlements are written in German and the most important of them have the Hungarian name too. We have identified on this map a number of 23 cities that can be found on actual maps.



Figure 2. "Neue Kriegs Kharthe von Siebenburgen, Moldau, Walachey", J.G. Schreiber, 1730

In the second map we can find Transylvania in the center (figure 2). After observing the aspect of the network of meridians and parallels we can assume that this map is made in a conic projection. In the top left corner, the map has a graphic scale divided in German and Hungarian miles, indicating that the map scale is approximately 1:3.500.000. We have identified 13 cities that can be found today on the territory of Transylvania.

Our study ends with a map published in 1862 that illustrates the Grand Duchy of Transylvania (figure 3). The difference between this map and the first one is that this map is realised based on precise measurements during one of the well-known surveys of the Austro-Hungarian Empire. In the right side of the map is a legend that provides us information about the symbols used by the author, such as: infrastructure, human settlements, relief, hydrographic network and also natural resources. The scale of the map is 1:200 000.

We have also included in our study an actual map of Romania, which is the base of planimetric accuracy comparison.

In order to make possible the study of precision, all the maps have to be made in the same projection and the same coordinate reference system.

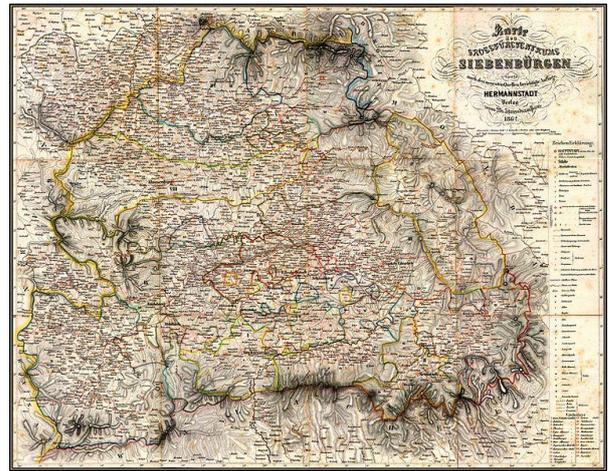


Figure 3. "Karte des Grossfürstentums Siebenbürgen", 1862

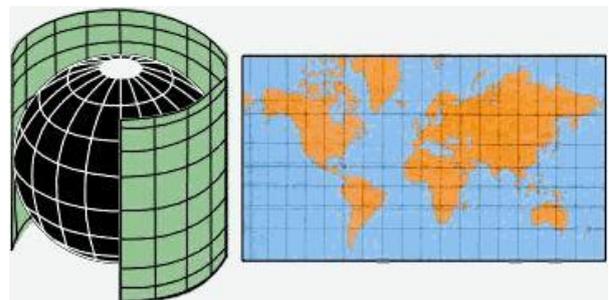


Figure 4. Cylindrical projection

The Mercator projection is a cylindrical projection introduced in 1569 by cartographer Gerardus Mercator. The Earth's surface is projected onto the surface of a cylinder which then carries out the plan by cutting cylinder after one of its generators (figure 4). The cylinder axis coincides with the poles axis. We approximate the Earth with a sphere of radius 6.375.000 meters and the cylinder being tangent at this sphere on equator. All the maps have been transformed in Mercator projection using formulas:

$$\begin{cases} X = \frac{\alpha}{0.4342945} \lg \left[\operatorname{tg} \left(45^\circ + \frac{\varphi}{2} \right) \right] \\ Y = \alpha \lambda \\ \alpha = R \cos \varphi_k ; \varphi_k = 0 \end{cases} \quad (1)$$

Using formulas (1) we have calculate rectangular coordinates (X, Y) for each node of the network of meridians and parallels. Then we have represented these points using AutoCAD. Based on these points we have georeferenced all maps. In this way the meridians and parallels networks have become rectangular. The georeferencing was realised using Autodesk software, AutoCAD and Raster Design.

In order to bring the maps in the same coordinate reference system we have applied the Helmert transformation with 4 parameters, consisting in a translation on the North axis (OX), a translation on the East axis (OY), a rotation ε and a scale factor m .

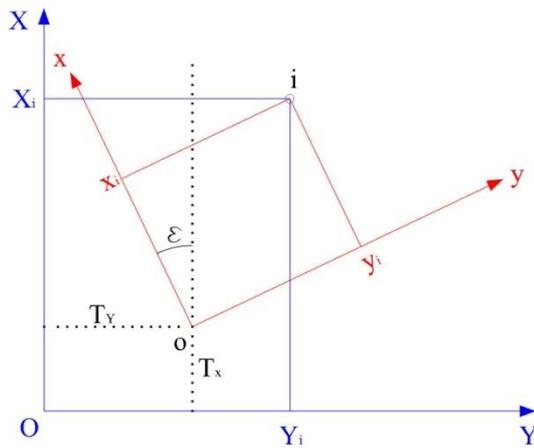


Figure 5: Helmert transformation with 4 parameters

In fig. 5, XOY is the system of the actual map and xoy is the system of each old map.

With notations: $a = m \cos \varepsilon$, $b = m \sin \varepsilon$, $c = T_x$, $d = T_y$, the formulas of transformation from a system to another become:

$$\begin{cases} X = ax - by + c ; \\ Y = bx + ay + d ; \end{cases} \quad (2)$$

For each point i that have coordinates in both systems we can write 2 equations:

$$\begin{cases} V_{X_i} = a x_i - b y_i + c - X_i \\ V_{Y_i} = b x_i + a y_i + d - Y_i \end{cases} \quad (3)$$

In the (3) formulas the corrections V_{X_i} and V_{Y_i} represent the comparison basis for accuracy of the cities.

RESULTS AND DISCUSSIONS

For each map the Helmert transformation was made using 10 major common cities: Alba Iulia, Bistrița, Brașov, Cluj, Dej, Făgăraș, Mediaș, Sebeș, Sibiu, Sighișoara.

After applying the Helmert transformation in the Mercator's map, "Transsylvania", we have obtained the following parameters:

$T_x = 150\,432$ m, $T_y = 2\,577\,483$, rotation $\varepsilon = 40^\circ$ and scale factor $m = 0.976302$. The East axis translation, T_y , corresponds to the

difference of longitude between the prime meridian of the actual map, Greenwich, and the prime meridian used by Mercator. Using the (1) formulas we have determined that the origin on longitude on Mercator's map is $23^\circ 09'$ West from Greenwich meridian, passing through Cape Verde Islands.

Applying the transformation parameters we get the results presented in Figure 6:

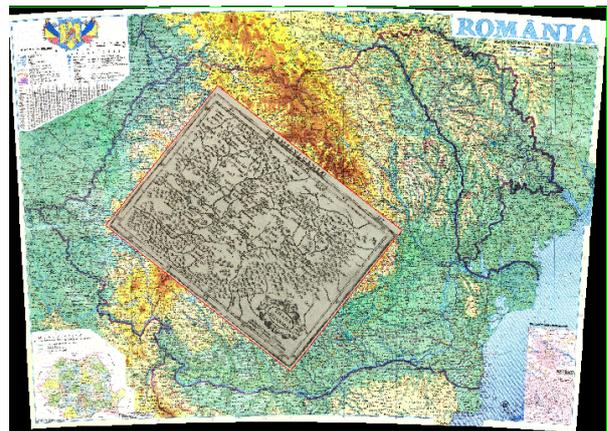


Figure 6. Transformation parameters for the map "Transsylvania"

Table 1. Planimetric accuracy for the cities on the map "Transsylvania"

| City | s [km] |
|------------|--------|
| Alba Iulia | 22.9 |
| Bistrița | 29.6 |
| Brașov | 25.8 |
| Cluj | 5.8 |
| Dej | 20.9 |
| Făgăraș | 24.6 |
| Mediaș | 23.8 |
| Sebeș | 21.2 |
| Sibiu | 17.3 |
| Sighișoara | 10.7 |

As we can observe in table 1, the average position error on this map is about 20 kilometers.

For the second map, made in 1730, we have obtained the following parameters: translation on the North axis: $T_x = 62\,338$ m, translation on the East axis: $T_y = 2\,513\,815$ m, rotation: $\varepsilon = 23^\circ 26'$ and the scale factor: $m = 1.104286$ (Figure 7).

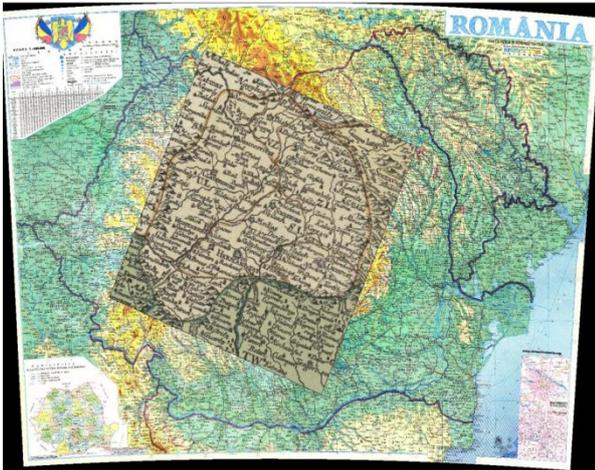


Figure 7. Transformation parameters for the map made in 1730

Table 2. Planimetric accuracy for the cities of the map „Neue Kriegs Kharte”, 1730

| City | s [km] |
|------------|--------|
| Alba Iulia | 2.5 |
| Bistrița | 23.5 |
| Brașov | 12.2 |
| Cluj | 21 |
| Dej | 6.3 |
| Făgăraș | 17 |
| Mediaș | 3.2 |
| Sebeș | 18.4 |
| Sibiu | 4.9 |
| Sighișoara | 19.2 |

In Table 2 is presented the position errors for the most important cities.

The average position error on this map based on table 2 is 12.82 kilometers.

On the last map which was made based on precise observations, we have obtained the following parameters: $T_x = -2740$ m, $T_y = 1$

971 239 m, rotation $\varepsilon = -00^{\circ}51'$ and scale factor $m = 0.994199$ (Figure 8).

Based on the Ty translation results that this map uses the Ferro meridian which passes through the Canary Islands.

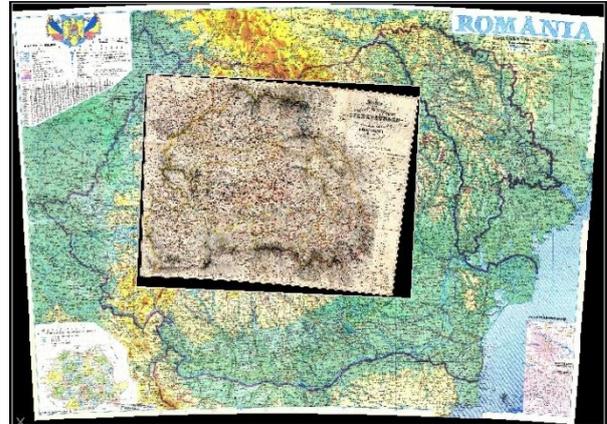


Figure 8. Transformation parameters for 1862 map

Table 3. Planimetric accuracy for the cities of the map „Karte des Grossfürstentums Siebenbürgen“, 1862

| City | s [km] |
|------------|--------|
| Alba Iulia | 3.5 |
| Bistrita | 4.6 |
| Brasov | 1.2 |
| Cluj | 1.8 |
| Dej | 4.8 |
| Fagaras | 1.3 |
| Medias | 1.3 |
| Sebes | 2.6 |
| Sibiu | 1.3 |
| Sighisoara | 2.8 |

On this map the average position error for those 10 cities is 2.52 kilometers, as we can see in table 3.

CONCLUSIONS

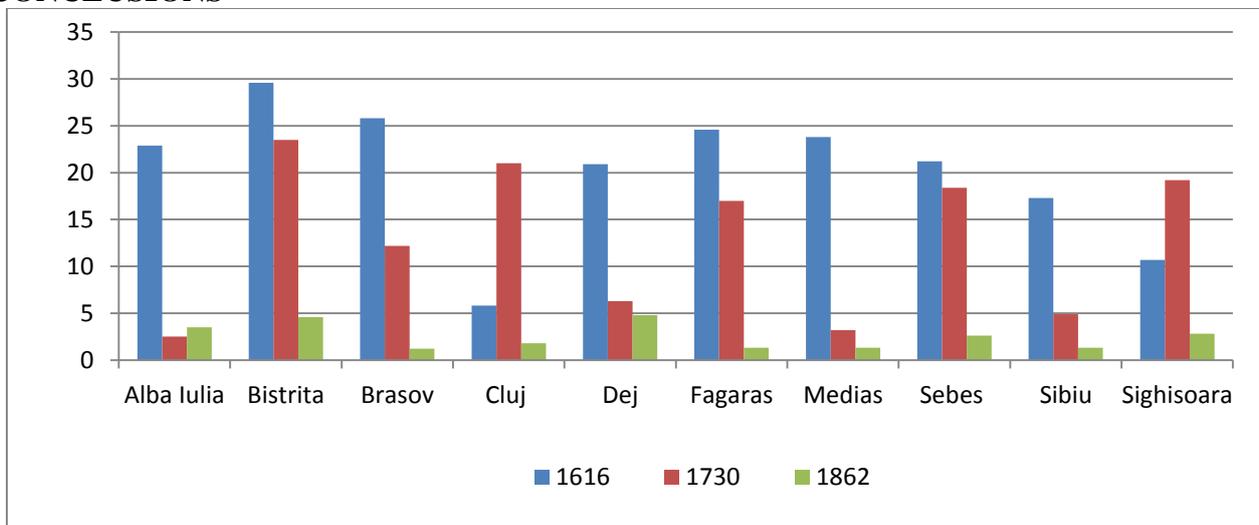


Figure 9: Planimetric accuracy for 10 most important cities



Figure 10. Average position errors



Figure 11. Rotation angle

As we can see in the Figures 9 and 10, the map made in 1862 is by far the most accurate and between this map and the modern maps it is a significant difference. This is due to precise surveys.

For the Mercator's map, the average position error is about 20 kilometers and judging by the instruments and the methods that he used at that time, we can consider that his map is still a very important tool in analyzing the historical and geographical landscape of Transylvania of 17th century. Considering that the scale of this map is about 1: 2.000.000, a graphic error of 1 millimeter represents 2 kilometers. That means

that for those times, the Mercator's map was one of the most accurate maps of Transylvania. For Schreiber's map made in 1730, as we can see in the fig. 10, the average error is about 13 kilometers. In the fig. 9 we can see that the errors are not uniform. Some cities such as Alba Iulia, Dej, Mediaș and Sibiu have small errors, while other cities like Bistrița and Cluj have errors bigger than 20 kilometers. In that time the precise surveying were barely started so we can assume that those measurements were not made in all those cities.

In the fig. 11 it is represented how the abnormal orientation of those three maps is changing. The orientation of the map made in 1862 is almost identical with modern maps orientation.

The rotation angle can also be an argument for the big errors which occur on the first two maps and as we observed once the rotation angle decreases the translation on North axis decreases too.

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