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COMPARATIVE ANALYSIS OF ARC-TO-CHORD CORRECTIONS BETWEEN THE GAUSS-KRUGER PROJECTED SYSTEM AND THE STEREOGRAPHIC 1970 PROJECTED SYSTEM FOR THE SAME SIDE OF TRIANGULATION NETWORK

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Abstract

The study conducted in this paper regards the comparative analysis of arc-to-chord correction between the Gauss-Kruger projected system and the Stereographic 1970 projected system. The correction is due to the fact that the measured direction between two points is actually a curved line, on the surface on a body such as an ellipsoid, that passes through these points. When projected onto a plane, the geodetic direction looks like an arc not a straight line. The angle that we compute from field notes is defined by the difference between two measured directions. Thus, the computed angle differs from the plane angle that we have to use when working with the State Plane Coordinate System. On the basis of the simple relation between spherical excess and arc to chord correction, formulas to compute the arc to chord correction for different map projections can be derived. In essence, research team aims to highlight the arc-to-chord differences obtained comparing the two projected system taken in consideration.

***Key words**: arc-to-chord correction, Gauss-Kruger, spherical excess, Stereographic 1970, triangulation network*

INTRODUCTION

Arc-to-chord correction become significant and they have to be determined for the II[nd] and III[rd] order networks, that's why we chosed points from the III[rd] order network of the Cluj county for the comparative study.

The directions reduction for the projection plan was realized for the Stereo 70 projection plan and also for Gauss Kruger.

The arc-to-chord correction- Stereo 70

Regarding the compute of arc-to-chord correction for Stereo 1970 projection plan, the operation is applied to azimuthal directions measured in the geodetic triangulation network and has to precede the rigorous compensation works. Each direction reduced to the ellipsoid measured from "i" station to "j" point from geodetic network, will receive a δ_{ij} correction with the value depending by the visa lenght, by orientation and by its distance from the origin of the xOy system axes . (Munteanu, 2003)

Regarding the deduction of a compute formula, in the stereographic projection plan we considered the geodetic points: 1 (x1,y1) and 2(x2,y2). The geodetic line 1-2 which joins the points to the ellipsoid has a plane image and a curve, concaved to the origin of the xOy axes system. In its extremites, in 1 and 2 points, the curve makes with its chord (Fig.1) the δ12 and δ21 small angles,which represent the arc-to-chord corrections of the directions to the projection plan.

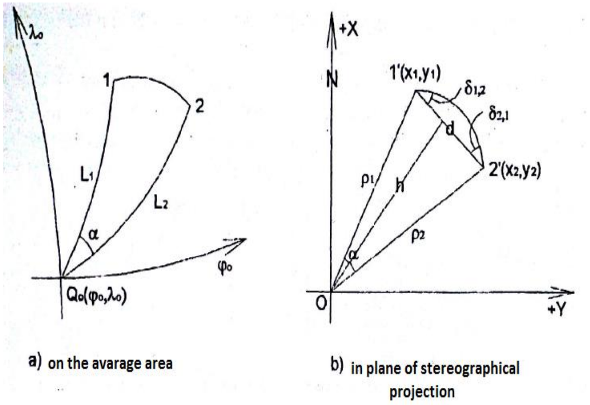

Figure 1. The Arc-to-chord correction for the Stereographical 1970 plan projection

The arc-to-chord corrections (δ_{12} and δ_{21}) represent the angles formed by the curved geodetic line with the chord which joins the end points of geodetic line. The plane image of the geodetic line is concaved towards the projection pole, which also represent the origin of the coordinates axes system. The arc-to-chord angles are positive when the passing from the curved line to chord is clockwise, and are negative when the passing is counter clockwise. (Moldoveanu, 2004).

Arc-to-chord corrections- Gauss Kruger

The geodetic lines from the ellipsoid, particulary the geodetic triangle sides from the ellipsoid are represented in the Gauss projection generally by curves with concavity to the axial meridian. In the two extreme points of geodetic line, the curve and its chord makes each one a small angle δ_{12} and δ_{21} (Fig. 2), representing the arc-to-chord corrections, for the Gauss projection plan.

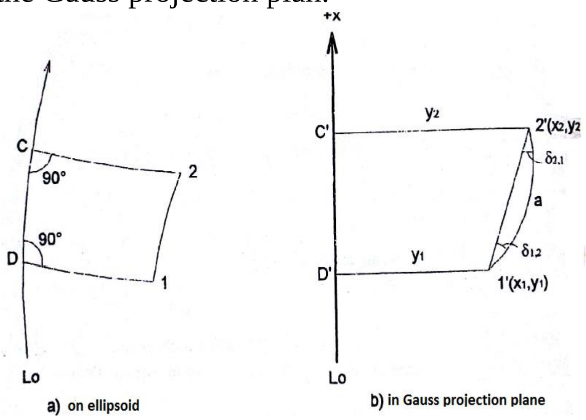

Figure 2. The Arc-to-chord correction for the Gauss - Kruger plan projection

If the asimutal surveys realized in the geodetic triangulation network have to be performed in the Gauss projection plan, then the surveyed asimutal directions have to be reduced to the plan of this projection system,
applying to each direction one correction compute with the specificy formulas of projection. (Palamariu et al., 2002)
The computed formulas for the reduction of directions to the Gauss projection plan are
different from a geodetic triangulation order to another, according to the neccesary precision.
For the IIIrd order networks, we consider the 1`(x1,y1) and 2`(x2,y2) points – the plane images of the points 1 and 2 from ellipsoid, and the curve 1'a2' – the plane image of the respective geodetic line. The C' and D' points are the perpendicular lines from 1' and 2' on the Ox axis. The δ_{12} and δ_{21} angles represent the reduction corrections of the directions to the gauss projection plan. (Ortelecan, 2006).

MATERIALS AND METHODS

In determining the arc-to-chord corrections were used formulas related to the spherical excess and other items that will be presented below.
Because the projection of the line, sum of the angles of the plane figure is equal to the sum of the angles corresponding to the ellipsoid (sphere of average radius). So we can write:

$$200^g + |\delta_{12}| + |\delta_{21}| = 200^g + \varepsilon$$
$$200^g + 2\delta = 200^g + \varepsilon$$
CASE STEREO(1)

$$400^g + |\delta_{12}| + |\delta_{21}| = 400^g + \varepsilon$$
$$400^g + 2\delta = 400^g + \varepsilon$$
CASE GAUSS (2)

(1) In the absolute value (δ), the correction is equal to half of ε spherical excess of the triangle formed by the station point, the target point and pole projection Q0

$$2\delta = \varepsilon$$

$$\delta = \frac{\varepsilon}{2}$$

$$\varepsilon^{cc} = \rho^{cc} \frac{S}{R^2},$$

where: ε - spherical excess
S – spherical triangle area
R – radius of the sphere

ρ – conversion factor from radians to seconds

The area of the triangle is determined by the determinant that contains the coordinates of the triangle tops and unity. The spherical triangle area can be assimilated with the plan triangle area. (Ortelecan, 2006)

.

$$S_1=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}=\frac{1}{2}(x_1\times y_2\text{-}x_2\times y_1)$$

Based on formulas (area, avarage radius) we can obtain the arc-to-chord corrections formula:

$$\delta=\frac{\rho^{cc}}{4R^2}\times\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

(2) For the spferical excess is well known the general formula:

$$\varepsilon^{cc}=\rho^{cc}\frac{S}{R^2},$$

where:

S- quadrilateral 12CD area on the ellipsoid

R is the average radius of curvature of the ellipsoid in the quadrilateral portion considered.

Since the spherical excess is relatively small, the compute of S area on the ellipsoid can replace the trapeze area S 'Plan 1' 2 `C` D `.

Arc-to-chord values are deteminated with the following formula:

$$\left|\delta_{12}^{cc}\right|=\left|\delta_{21}^{cc}\right|=\frac{1}{2}\varepsilon^{cc}=\frac{\rho^{cc}}{2R^2}\times\frac{(y1+y2)}{2}\times(x2\text{-}x1)$$

Direction determination was done using Zeiss Theo 010 (Fig. 3).

Tachymeter-theodolite Zeiss Theo 010 is a precision theodolite, recommended for all kind of geodetic-topographic projects that admits a mean square error of ± 4 ^{cc} (1,5^{cc}), for a single measured direction in both telescope positions. Main usages are: triangulations on IInd, IIIrd and IVth grades, precision pollinations done day or night, tracing constructions. For horizontal and vertical circle readings, it is used an microscope with optical micrometer with coincidence and reading is done using an optical device.

Figure 3. Tachymeter-theodolite Zeiss Theo 010

RESULTS AND DISCUSSIONS

The case study on the design of azimuthal directions in Stereo 70 projection plane was performed in three points corresponding to triangulation network of Cluj-Napoca. The following points were studied: Dl.Hoia, La Pipa, Dl.Sopor (Figure 2). The coordinates of points in the Stereo 70 projection plane are presented in Table 1. They were reduced to the projection plane - Table 2.

Table 1. Stereo 1970 Coordinates

Point	X	Y
Dl. Hoia (47)	**586465.38**	**388398.37**
La Pipa (42)	**590814.83**	**398776.73**
Dl. Sopor (48)	**584577.58**	**397156.93**

Table 2. Stereo 1970 Coordinates - Coordinates with origin in projection pole

Point	X	Y
Dl. Hoia (47)	86465.38	-111602
La Pipa (42)	90814.83	-101223
Dl. Sopor (48)	84577.58	-102843

To determine the corrections of arc-to-chord reductions for Gauss-Kruger system it was necessary to transform the Stereo 1970 coordinates to Gauss-Kruger coordinates.

First of all, it was realised the transformation of Stereo coordinates to geographical coordinates. This was done in two steps:

- Step 1 is the transformation of geographical coordinates of secant plane to tangent plane: scale is changed by multipling with “c” coeficient, which is named reversion to normal scale factor

- Step 2 represents the transformation of stereographic coordinates of tangent plane to geographical coordinates B, L of Krasovski 1940 ellipsoid. It is solved using formulas with constant coeficients.

Obtained values are presented in Table 3.

Table 3. Geographical Coordinates

Point	B	L
47	**46.1006**	**23.3222**
42	**46.4833**	**23.4025**
48	**46.451**	**23.3916**

Next the geographical coordinates were transformed into Gauss - Kruger coordinates.

Knowing the geographical coordinates of a point of the rotation ellipsoid (B, L), the x and y plane Gauss - Kruger coordinates of that point were calculated.

The method of constant coefficients was applied, and then Gauss coordinates of that three points were obtained.- Table 4.

Table 4. Gauss-Kruger Coordinates

Punct	x	y
47	**5184721.209**	**4693965.312**
42	**5189592.408**	**4704110.736**
48	**5183280.051**	**4702809.416**

Next, the compute of the of arc-to-chord corrections in Stereo projection plane and in Gauss-Kruger plane was done.

<u>Reduction of azimuthal directions to Stereo projection plan</u>

The compute of the arc-to-chord corrections, as well as compute of liniar deformation modulus was realised using the coordinates of that points that have the origin in projection pole.

Based on the presented formulas we determined the of arc-to-chord corrections for measured directions.

Control practical rule:

In every geodetic triangle, the arc-to-chord correction sum to the projection plan for those three angles have to be equal with the value of spherical excess for the respective triangle took with changed sign.In the figure 5 we can observe the β_1, β_2 and β_3 angles equals with those three angles from the ellipsoid.

$$\beta_1+\beta_2+\beta_3 = 200^g+\varepsilon$$

β_1\`, $\beta_2$\` si β_3\` angles, between the sides which joins the triangle’s tops are reduced to the projection plan. (Ortelecan, 2006)

The arrangement of the three points that were studied in the geodetic triangulation network of Cluj county is presented in Figure 4.

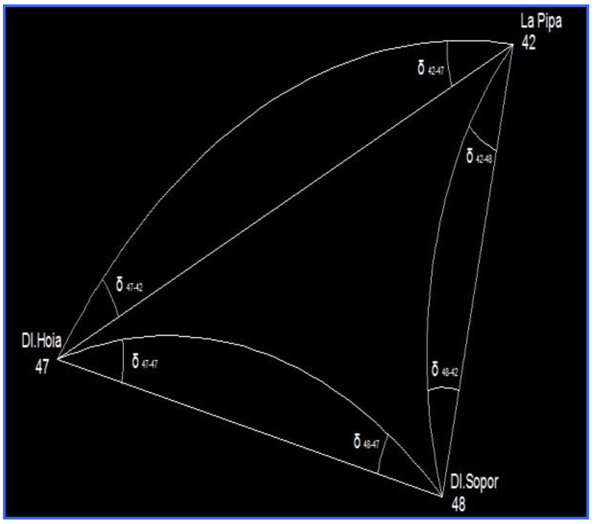

Figure 4. Analised geodetic triangle

Using the area of the plane triangle we determine the value of sferical excess on triangle.

$$\varepsilon = \frac{636620}{2\cdot 6378957^2}\cdot\begin{vmatrix} 86465.38 & -111601.63 & 1 \\ 90814.83 & -101223.27 & 1 \\ 84577.58 & -102843.07 & 1 \end{vmatrix} = 0,451264$$

Values of arc-to-chord corrections are presented in Table 5.

Table 5. Values of arc-to-chord corrections

Name of correction	Value of correction
δ_{47-42}	5.40844373
δ_{42-47}	-5.4084437
δ_{47-48}	2.13803557
δ_{48-47}	-2.1380356
δ_{48-42}	3.04477637
δ_{42-48}	-3.0447764

$\beta_1' + \beta_2' + \beta_3' = 200^g + \varepsilon$

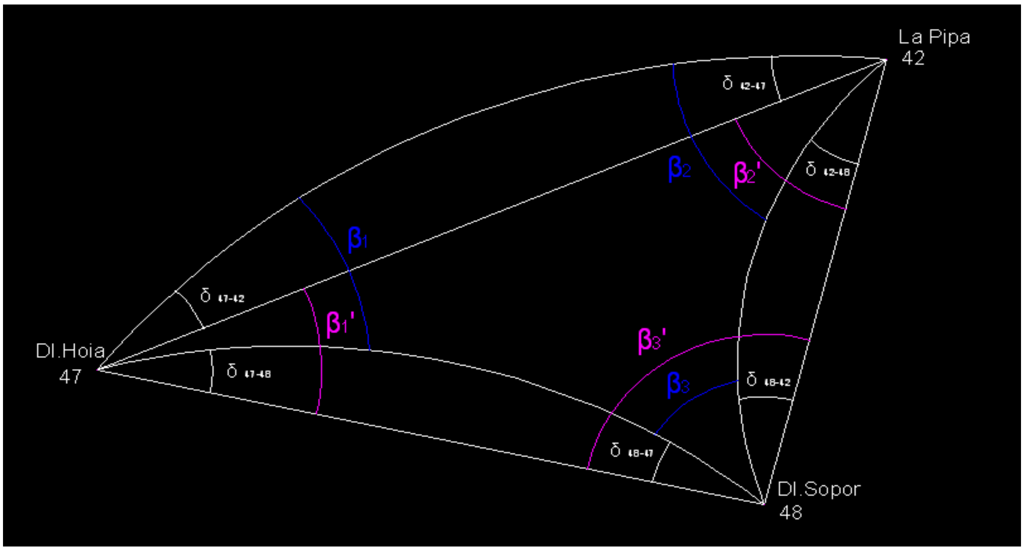

Figure 5. The control of arc-to-chord corrections to the projection plan

We note α_m = the measured direction, α_r = the reduced direction at the projection plan and δ= the reducing correction at the projection plan, generaly it can be write like this:

$$(\alpha_r)_{ij} = (\alpha_m)_{ij} + \delta_{ij}$$

Apllying the relation for all the triangle's tops, considerated, like stations, we obtain:

$$\beta_1' = \beta_1 + (\delta_{47-48} - \delta_{47-42})$$
$$\beta_2' = \beta_2 + (\delta_{42-48} - \delta_{42-47})$$
$$\beta_3' = \beta_3 + (\delta_{48-42} - \delta_{48-47})$$

By the direction coefficient difference it has been established the angular corrections for the reducing in the projection plan.(Table 6) :

Table 6. Angular correction

Angular corrections	Value
$\delta_{47-48}-\delta_{47-42}$	-3.27041
$\delta_{48-42}-\delta_{48-47}$	5.182812
$\delta_{42-47}-\delta_{42-48}$	-2.36367
[ ] = ε^{cc}	-0.45126

After all the efectuated computes, the truth of the affirmation mentionated above is confirmed.

<u>Reduction of azimuthal directions to Gauss - Kruger projection plan</u>

The area of the triangle is determined by the determinant that contains the Gauss coordinates of the triangle tops and unity.

	5184721.209	**4693965.312**	1
det=	**5189592.408**	**4704110.736**	1
	5183280.051	**4702809.416**	1

Applying the formula $\varepsilon^{cc} = \rho^{cc}\dfrac{S}{R^2}$, we obtain:

ε^{cc} =	0.4512981

The arc-to-chord reduction is performed taking into account the following principles:

1. Ordinate values will be reduced by canceling the meridian axial first digit indicating the time zone and by decreasing the amount of 500 000 000 m (table 7).

Table 7. Reduced Gauss-Kruger coordinates

Punct	x	y
47	**5184721.209**	**193965.312**
42	**5189592.408**	**204110.736**
48	**5183280.051**	**202809.4165**

2. The numerical values of the abscissa and ordinate respectively environments are introduced in formula in meters with 3 decimal digits.

3. The numerical values of arc-to-chord corrections are expressed in third order geodetic points with two decimal places given the precision of determination of ± 0 "1.

4. The numerical values of the corrections calculated in terms of reducing directions both ways positive or negative depending on the difference of the abscissae.

The arc-to-chord corrections values obtain for Gauss - Kruger projection plan are presented in Table 8.

Table 8. Values of arc-to-chord corrections

Name of correction	Value of correction
δ_{47-42}	-7.583907202
δ_{42-47}	7.583907202
δ_{47-48}	2.236394689
δ_{48-47}	-2.236394689
δ_{48-42}	-10.04555764
δ_{42-48}	10.04555764

CONCLUSIONS

The arc-to-chord corrections precede the offset angle triangulation networks, given the fact that by applying these corrections spherical excess is eliminate.
After the calculations for the two projection systems we notice higher values of arc-to-chord corrections for Gauss - Kruger system.
Arc-to-chord corrections size is influenced by the length of the chord visa, the distance from the pole projection and orientation visa.
Values of arc-to-chord corections become significant discount to be applied to the order triangulation networks III, II, I.

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