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ESTABLISHING THE SPHERICAL EXCESS FOR THE GEODETIC NETWORKS OF THE THIRD ORDER

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Abstract

The object of our study is the spherical excess and the national geodetic network for the city of Cluj-Napoca. As a proof of the earth's rotundity, many place great reliance upon what is called the "spherical excess," which has been observed on making trigonometric observations on a large scale. The angles taken between any three points on the surface of the earth by the theodolite are, strictly speaking, spherical angles, and their sum must exceed 180 degrees; and the lines bounding them are not the chords as they should be, but the tangents to the earth. This excess is inappreciable in common cases, but in the larger triangles it becomes necessary to allow for it, and to diminish each of the angles of the observed triangle by one-third of the spherical excess. In other words, the spherical excess is represented by the difference between the sum of the angles of a spherical triangle and the sum of the angles of a plane triangle. The national geodetic network and the triangulation network represent the fixed points which form the base of all leveling procedures. Considering the distance between the points and the measurements accuracy, the points that form the geodetic network are classified in five categories: first order: the points are situated between 20-60 km, average 30 km, second order: the tips of the triangles are intercalated in the points of first order at distances between 10-20 km, average 15km,third order: the points are situated inside of the triangles of the second order at distances between 5-10 km, average 7 km, forth order: the points are situated inside of the triangles of the third order at the average distance of 3 km, fifth order: the points are situated inside of the triangles of the forth order at the average distance of 1.5 km. In our project we are using the points of the third order to determine the spherical excess in the city of Cluj-Napoca.

***Key words**: spherical excess, spherical triangle, triangulation network, area, correction.*

INTRODUCTION

Distance and azimuthal observations made on the topographic surface between the points of the triangulation network are projected on the reference surface (ellipsoid or sphere of average radius).Thus, the lines which join the points of the triangulation network appear as curves, determining on the reference surface spherical triangles (Munteanu, 2003; Ortelecan and Sălăgean, 2014).

According to spherical trigonometry, a spherical triangle is defined as a figure formed on the surface of a sphere by three great circular arcs (section of a sphere that contains a diameter of the sphere) intersecting pairwise in three vertices. The spherical triangle is the spherical analog of the planar triangle, and is sometimes called an Euler triangle (Figure 1).

As opposed to the geometric condition in a plane triangle, according to which the sum of its angles is 200 degrees, in a spherical triangle the sum of the angles, unaffected by measurement errors, will be equal to 200 degrees plus the spherical excess (Dima, 2005; Ortelecan, 2006).

In other words, the spherical excess is the amount by which the sum of the three angles of a spherical triangle exceeds two right angles.

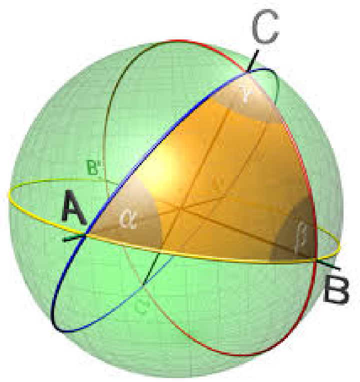

Figure 1. Spherical triangle

MATERIALS AND METHODS

In order to calculate the spherical excess, we used the triangulation network for the city of Cluj-Napoca and processed the information using a CAD software and Microsoft Excel.

As method, we applied the reduction of the directions to the plane of the Stereographical 1970 Projection. This calculus operation is applied to the azimuthal directions measured in the geodetic triangulation network.

In principle, every direction that is reduced to the reference surface measured between two points will receive a correction δ, whose value depends on the length of the sight, orientation and the distance towards the centre of the axis system xOy (Figure 2).

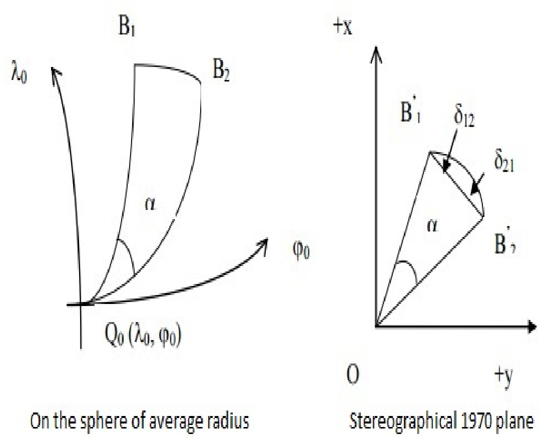

Figure 2. Reduction of the directions to the plane

RESULTS AND DISCUSSIONS

The general formula for calculating the spherical excess is:

$$\varepsilon = \rho * \frac{S}{R^2}$$

where: S – spherical triangle area;

R – spherical radius.

Other formulas used in this study:

$$S_1 = \frac{1}{2} * \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\delta = \frac{\varepsilon}{2}$$

$$\delta_{b-a} = -\delta_{a-b}$$

We picked three points from the triangulation network from our city to exemplify the calculus of the spherical excess regarding the third order points (Figure 3).

Table 1 - The point coordinates

Point number	x	y
40	592373.84	389535.84
42	590814.83	398767.73
47	586465.38	388398.37

Since the correction has small values (seconds or tens of seconds) ,it is enough to know the S area , with approximation , to calculate it.

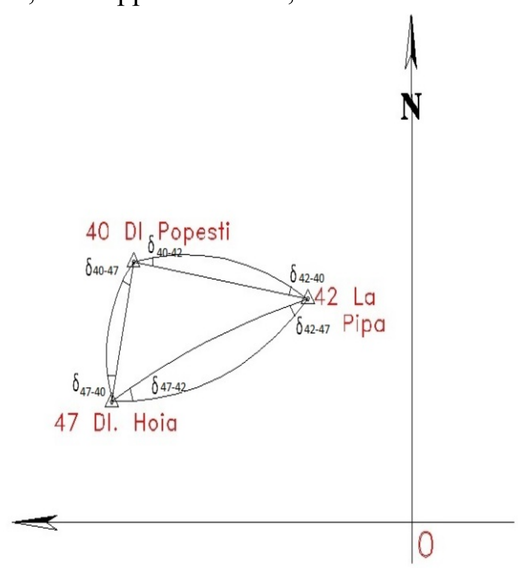

Figure 3 - The points taken into study

So, the spherical triangle area can be replaced with the area of the plane triangle, which we calculate using a determinant that has as elements the coordinates of the tips of the triangle and unity:

$$S1 = \frac{1}{2}\begin{vmatrix} 592373.84 & 389535.84 & 1 \\ 590814.85 & 398767.75 & 1 \\ 586465.38 & 388398.37 & 1 \end{vmatrix}$$

To calculate the spherical excess, we used the general formula:

$$\varepsilon = 636619.7724 * \frac{28159789.98}{40691087304683.6} = 0.44056525$$

To eliminate the spherical excess ,we used the reduction of the directions to the plane of the Stereographical 1970 Projection to calculate the δ correction for reduction to the projection plane.

For finding δ_{47-40} in Δ47-40-0

$$S = \frac{1}{2} * \begin{vmatrix} 586465.38 & 388398.37 & 1 \\ 592373.84 & 389535.84 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

S=813874728.7

$$\varepsilon = 636619.7724 * \frac{813874728.7}{40691087304683.6} = 12.7332243$$

$$\delta_{47-40} = \frac{12.73322437}{2} = 6.366612185$$

For finding δ_{40-42} in Δ40-42-0

$$S = \frac{1}{2} * \begin{vmatrix} 592373.84 & 389535.84 & 1 \\ 590814.83 & 398767.73 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

S=3038010200

$$\varepsilon = 636619.7724 * \frac{3038010200}{40691087304683.6} = 47.5302453$$

$$\delta_{40-42} = \frac{47.5302453}{2} = 23.7651227$$

For finding δ_{47-42} in Δ47-42-0

$$S = \frac{1}{2} * \begin{vmatrix} 586465.38 & 388398.37 & 1 \\ 590814.83 & 398767.73 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

S=2195975681

$$\varepsilon = 636619.7724 * \frac{2195975681}{40691087304683.6} = 34.3564557$$

$$\delta_{47-42} = \frac{34.3564557}{2} = 17.1782278$$

Checking rule:

In every triangle which belongs to the geodetic network from the plane of a projection, the sum of the correction for reduction to the projection plane of the three angles of the triangle, should be equal with initial –ε spherical excess.

$$-\varepsilon = (\delta_{47-42} - \delta_{47-40}) + (\delta_{40-47} - \delta_{40-42}) + (\delta_{42-40} - \delta_{42-47}) = -0.44056525$$

In our research, we found a particular case in which the correction and the spherical excess is almost null .

Using our data, we found out the following results throught the method :

Table 2 - Point coordinates

Point number	x	y
213	588914.72	389913.78
238	583061.97	397113.41
0	500000	500000

$$S = \frac{1}{2} * \begin{vmatrix} 588914.72 & 389913.78 & 1 \\ 583061.97 & 397113.41 & 1 \\ 500000 & 500000 & 1 \end{vmatrix}$$

S=2077019.28

$$\varepsilon = 636619.7724 * \frac{2077019.28}{40691087304683.6} = 0.03249536$$

CONCLUSION

The tolerance with which the checking rule must be satisfied depends on the triangulation order: 0.03” for the third order triangulation.

As we found out on the direction 213-238-0,the spherical excess is almost null ,this happens when the station point , sight point and the origin of the axis are collinear.

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REFERENCES

Dima N., 2005, Geodezie, EdituraUniversitas, Petroşani

Munteanu C., 2003, Cartografiematematică, Editura Matrix Rom, Bucureşti

Ortelecan M., Sălăgean T., 2014, Geodezie lucrări practice, Editura Risoprint, Cluj-Napoca

Ortelecan M., 2006, Geodezie, Editura Academic Pres, Cluj-Napoca