

## SETTING THE REDUCE CORRECTIONS AT THE CHORD, FOR DIRECTIONS IN THE CASE OF THE 1970 STEREO PROJECTION

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### Abstract

The first step to be run as part of the compensation is to determine preliminary coordinates. They are determined with a precision low accuracy generally depends on the purpose and length of the sides of the network considered. Because the projection system used officially in Romania is 1970 stereographic system for processing observations is usually a two-dimensional system still will consider that this plan is the reference surface which will reduce geodetic observations. If the geodesic network of the distance measurements were made, to be reduced from the reference surface selected. After being corrected physical (generally modern tools to measure distances apply this correction automatically) distances measured to be applied, in order, the following discounts: Reducing the chord; Reduction of the reference ellipsoid surface; Reduction plan stereographic projection in 1970. This stage of preliminary processing geodetic observations and reduction in the chosen reference surface is considered performed by the user, for which the program will be introduced reduced to the reference surface observations.

**Key words:** Reducing the chord, compensation, projection system, topography

### INTRODUCTION

The 1970 stereographic projection is an azimuthal, perspective (using linear perspective laws) and determined (preserves angles) projection. The secant projection plan is being lowered by 3,502m as opposed to the tangent plane onto the equivalent sphere.

Almost the entire surface of Romania can be enframed in a circle with the radius of about 400 km. The centre of the circle is considered to be also the origin of the rectangular coordinate system XOY and it is situated on the north side of the town of Făgăraş, at the intersection of the 46° north latitude parallel and the 25° longitude meridian. This proves that the whole country is divided into four quadrants. The OX axis is orientated towards North, and the OY axis is directed towards East. In order to reduce the number of calculations, a false axis system is used whose origin point is translated, in the same plan, 500km towards South and West; this way all

the referential points in the false system will have only positive coordinates.

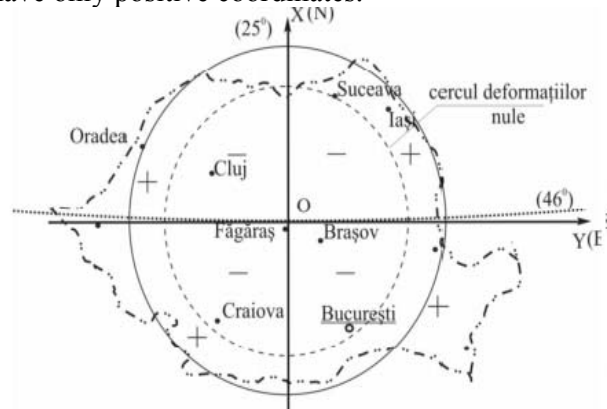


Figure 1 – The entire surface of Romania (Munteanu, 2003)

Reducing the projection plane directions is the operation of correcting the measured directions in the state geodesic network, through the application of angular corrections  $\delta$  called "corrections to reduce the chord". This operation is necessary because, in the plane of projection, flat images of GEODESIC triangles sides are not lines but are curved. To determine

the formula for the calculation of the corrections, we consider a spheric triangle, B1-B2-Q0 (B1, B2 are the extremities of a measured direction, and iar Q0 ( $\lambda_0, \varphi_0$ ) is the projection pole), on the sphere of medium RADIU R0 .

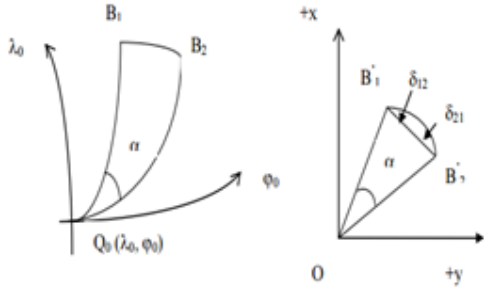


Figure 2 - The representation of the geodesic lines on the ellipsoid and on the projection plane (Palamariu, Padure, Ortelecan, 2002)

For the representation of spherical triangles in the plane, we consider the following properties of the stereographic projection: the projection is determined; the big circles passing through Q0 (verticals) are represented by line segments passing through the origin O; an arc will always be represent through an arc (the exceptions are the verticals).

The planar images of the tips of the spherical triangle are the points B1, B2 ' and ' O. The arcs of circle B1Q0 and B2Q0, belong to some verticals of the pole Q0, which are represented through the lines B1'O and B2 ' O, forming an angle  $\alpha$ , between them, equal to the one corresponding to the one on the sphere. The chord reduction refers to the deformation of the directions that were moved from the equivalent sphere on the plane of projection. All the directions from the sphere are projected as curved lines on the plane, with the exception of those which pass through the point opposite to the stereographic point S. These directions are projected as straight lines passing through the origin of the coordinate axes.

Any direction 1-2 from the sphere will be projected on the plan through a curved line, creating a spherical triangle with the directions O-1 and O-2 passing through the origin. Considering this, the only side of the triangle that is affected by the spherical excess is 1-2.

If the distance between 1 and 2 is sufficiently large (kilometre-order), we should take

intoaccount the size of the spherical excess, given by:

$$\varepsilon^{cc} = \rho^{cc} \frac{S}{R^2}$$

The size of the surface can be calculated using the formula:

$$S_1 = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = \frac{1}{2} (x_1 \times y_2 - x_2 \times y_1)$$

In these circumstances, the formula

$$\varepsilon^{cc} = \rho^{cc} \frac{S}{R^2} \text{ becomes :}$$

$$\varepsilon^{cc} = \frac{S}{2R^2} \rho^{cc} = \frac{\rho^{cc}}{4R^2} (x_1 y_2 - x_2 y_1)$$

Using the value in the previous formula, we can determine the azimuths from the plan, thus eliminating the effect of spherical excess. This helps us notice that, in the event of such determinations, the coordinates of the points 1 and 2 from the formula are the real ones, not the once affected by the translation of 500 km on the South and West direction.

## MATERIALS AND METHODS

For the calculation of the reduction at the chord corrections will take account the Figure 3.

The representation to be determined (to preserve angles in planes of projection)

Geodetic lines that pass through the pole projection (Q0) will be represented in the plane of projection by straight line segments passing through the origin of the axes.

Coastlines of great circles on the sphere that do not pass through the pole projection (Q0), will be represented by the arcs.

Based on these considerations, the projections in the plan projection directions will appear as shown in the Figure 4.

The representation is determined, so the sum of the angles of the figure of the plan shall be equal to the sum of the angles of the corresponding figure on the ellipsoid and results:

$$180^\circ + 2\delta = 180^\circ + \varepsilon$$

$$2\delta = \varepsilon$$

$$\delta = \frac{\varepsilon}{2}$$

(1)

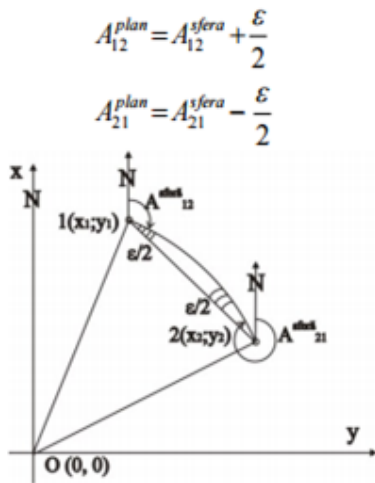


Figure 3 – Reducing the corrections at the chord

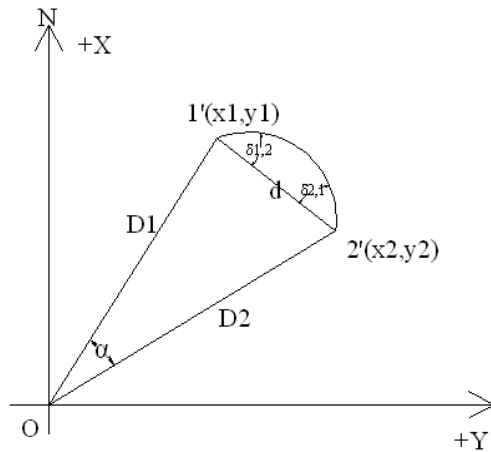


Figure 4 – Direction projected in the plan

In absolute value ( $\delta$ ), the correction shall be equal to half of the excess of a triangle formed by spherical  $\varepsilon$  from the target point and station pole  $Q_0$  of the projection. Calculating spherical excess is given by the relation :

$$\varepsilon^{cc} = \rho^{cc} \frac{S}{R^2} \quad (2)$$

Where:  $\varepsilon$  – spherical excess  
 $S$  – spherical triangle area  
 $R$  – the radius of the sphere  
 $\rho$  – the coefficient of transformation from radians in seconds.

The radius of the sphere is calculated according to the relationship :

$$R = \sqrt{MN} \quad (3)$$

where:  $M$  - the radius of the ellipse meridian  
 $N$  - the radius of curvature of the first vertical

$$M = \frac{a(1-e^2)}{W^3} \quad (5)$$

$$N = \frac{a}{W} \quad (6)$$

$a$  - large semimajor axis (Equatorial radius)

$e^2$  - first excentriciate

$$e^2 = \frac{a^2 - b^2}{a^2} \quad (7)$$

$b$  - small semimajor axis

$W$  - the auxiliary function

$$W = \sqrt{1 - e^2 \sin^2 B} \quad (8)$$

$B$  - latitude

Area of a triangle plane  $S_1$  can determine a determinant that contains the coordinates of the triangle and the peaks. The spherical triangle can assimilate with the triangle area plan.

$$S_1 = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = \frac{1}{2} (x_1 \times y_2 - x_2 \times y_1)$$

Replacing with the relationship (2) and (3) in relationship (1) is obtained:

$$\delta = \frac{\rho^{cc}}{4R^2} \times \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix} \quad (10)$$

## RESULTS AND DISCUSSIONS

The case study on the design of bearing in the directions of the Stereo 70 projection was done in three points of triangulation network relating to Cluj-Napoca. There were taken into study points like: Gradina Botanica, Chinteni și Calea Turzii. These are points of triangulation of the II, III and IV order.

Calculation of the corrections of the chord and the discount calculation global deformation response linear module is carried out with the coordinates of the points that have origins in the projection. In this case the coordinates will fall 1970 Stereo on the X and Y value of false origin 500,000 [m].

Table 1 - Stereo Coordonates

Stereo 1970 Coordonates		
Punct	X	Y
Gradina Botanica ( turn ) (803)	585444.580	392308.900
Chinteni (1102)	592195.620	391924.350
Calea Turzii (220)	582906.660	392258.000

Table 2 - Coordinates with origin in the projection pole

Coordinates with origin in the projection pole		
Punct	X	Y
Gradina Botanica ( turn ) (803)	85444.580	-107691.100
Chinteni (1102)	92195.620	-108075.650
Calea Turzii (220)	82906.660	-107742.000

According to the above relations are established the reduction at the chord corrections for the measured directions.

Table 3 - Corrections

Correction Name	Correction Value
$\delta_{803,1102}$	2.715103189
$\delta_{803,220}$	-1.086013276
$\delta_{220,1102}$	3.806277756
$\delta_{1102,220}$	-2.7151
$\delta_{220,803}$	1.086013
$\delta_{1102,803}$	-3.80628

On the basis of the area of the triangle plan are determined the calculation of spherical excess:

$$\varepsilon = \frac{636620}{2 \times 6379,391^2} \times \begin{vmatrix} 85444.580 & -107691.100 & 1 \\ 92195.620 & -108075.650 & 1 \\ 82906.660 & -107742.000 & 1 \end{vmatrix} = -0.0103$$

The differences of these directional coefficients have been determined for reducing the angular adjustments in the plane of projection:

Table 4 – Amount of correction

Angular correction	The amount of the correction
$\delta_{803,220} - \delta_{803,1102}$	-3.801116465
$\delta_{1102,220} - \delta_{1102,803}$	1.091174568
$\delta_{220,1102} - \delta_{220,803}$	2.72026448
[ ]	0.0103

From this table, it appears that the amount of angular corrections from a network of triangulation triangle is equal to the inverse of the spherical excess value.

The sign for the reduction at the chord corrections shall be determined according to the direction of travel from the curve of the geodesic lines to the chord. When the meaning is directly clockwise the correction will get positive and negative otherwise.

The establishment of the sign must be done with great care because a wrong choice of the sign inserts a double error correction for the wrong set.

## CONCLUSIONS

The calculation of the corrections to reduce chord must precede the offset angles in triangulation networks, considering the fact that by applying these patches eliminate spherical excess. Patch size reduction from the chord is influenced by the length of the visa, the distance from the pole and the orientation of the projection of the visa.

The direction of the approaching visa guidance from the station point to the center point of projection, the correction value of chord discount shrinks .

Corrections discount values to string become meaningful and must be applied for second-order triangulation networks III, II, I.

## REFERENCES

- Moldoveanu, C., Geodezie, Editura Matrix Rom, București, 2004
- Ortelecan, M., Geodezie, Editura AcademicPres, Cluj-Napoca, 2006
- Munteanu, Gh., Constantin, Cartografie matematică, Editura Matrix Rom, București, 2003
- Palamariu, M., Pădure, I., Ortelecan, M., Cartografie și cartometrie, Editura Aeternitas, Alba-Iulia, 2002