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MATHEMATICAL MODELING OF POLLUTANTS IN SEWAGE

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***Abstract*:**

Mathematical modeling of water movement and dispersion of pollutants in sewage, is a major environmental problem. Increasing the number of pollutants, concentration, appearance of new compounds with unknown side make this a major issue in the operation of sewer networks. This paper aims to pesente several aspects of the movement of water and pollutants in sewage.

Key words: *constant motion, equation of continuity, non-permanent moving, Navier-Stokes equations.*

INTRODUCTION

Fluid motion can be described quantitatively by mathematical expressions, which are based on three fundamental physical laws: the law of conservation of mass, with the aid of we obtain the continuity equation, Newton's Second Law, which we obtain the equation of dynamic equilibrium, called the Navier-Stokes equation, and the law of energy balance equation. In most cases the fluid motion equations and general dispersion equation are solved together, forming a system and the specific mass of the fluid depends on the pollutant concentration. To characterize the phenomenon of contaminant transport modeling is necessary to know several parameters: the distribution of velocities, pressure and temperature of the water, which they depend on the shape, on the flow and on the force field acting of the water. Pollutants must be identify and need to know the laws after they propagate, they move, react and then taken defensive measures and decrease the negative effects. Fluids are characterized by four components of pollutant transport: accumulation, convection, diffusion and chemical, biochemical, biological transformations occurring in the evolution of the prismatic channel.

MATERIALS AND METHODS

Pollutant dispersion is the result of the simultaneous action of a molecular diffusion phenomenon (exchange of molecules between layers of fluid) - advection due to the existence of speeds field which occurred the pollution. Transport and dispersion of pollutants in channels and pipes are in permanent and non-permanent movement. Permanent movement does not depend on time, only depend on the variable x. This movement is justified in rivers for low water periods, which have a relatively long course with high pollution.

If we replace c (concentration variable), with L (equivalent pollution variable), it will state the transport and evolution equation of the pollutant:

$$Dt\frac{d^2L}{dx^2} - v\frac{dL}{dx} - k_d L = 0 \quad , \qquad (1)$$

with boundary conditions:

x = 0, L = L_1;

x = L, L = L_2,

where L - CBO, Dt - dispersion coefficient; kd-coefficient of oxygen deficiency, x - distance from the origin section to the calculation section of water; L_1 and L_2 are known values, defined by the boundary conditions. Non-permanent movement depends on the time and space (variables) of the water flow, velocity and concentration of pollutants.

Q = Q (x, t) c = c (x, t),

where Q = water flow and c = concentration of the pollutant.

Concentration of pollutant transported by water currents in rivers and channels generally varies in time and space and does not affect water movement. Transport of pollutants in the water with average speed, involves networking between pollutant concentration and spatial characteristics - time, thus:

$$\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=Dt\frac{\partial^2 c}{\partial x^2}-kc, \quad (2)$$

c – concentration of the pollutant,
Dt - dispersion coefficient,
k – coefficient of proportionality,
where the initial conditions are:
t=0; $c=c_0$ (x) and boundary conditions:
$x=x_1$; $c=c_1(t)$; x=l; $c=c_2(t)$.

Numerical simulation of non-permanent movement of fluids is defined by the independent variables t and x, by taking steps Δx, Δt. Derivatives function (c) are presented in two computation schemes: (3)

1.explicit scheme:

$$\frac{\partial c}{\partial t}=\frac{c_j^{n-1}-c_j^n}{\Delta t};\frac{\partial c}{\partial x}=\frac{c_{j+1}^n-c_j^n}{\Delta x};\frac{\partial^2 c}{\partial x^2}=\frac{c_{j+1}^n-2c_j^n+c_{j-1}^n}{\Delta x^2}$$

2. implicit scheme: (4)

$$\frac{\partial c}{\partial t}=\frac{c_j^{n-1}-c_j^n}{\Delta t};\frac{\partial c}{\partial x}=\frac{c_{j+1}^{n+1}-c_j^{n+1}}{\Delta x};\frac{\partial^2 c}{\partial x^2}=\frac{c_{j+1}^{n+1}-2c_j^n+c_{j-1}^{n+1}}{\Delta x^2}$$

The fluid equations which are characterize the transport and dispersion of pollutants are the Navier-Stokes equations and the continuity equation. For real fluids, the specific weight is constant and based on mass forces, Navier – Stokes equations have the following expressions: (5)

$$X-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\Delta v_x=\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}+v_z\frac{\partial v_x}{\partial z}$$

$$Y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\Delta v_y=\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_y}{\partial y}+v_z\frac{\partial v_y}{\partial z}$$

$$Z-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\Delta v_z=\frac{\partial v_z}{\partial t}+v_x\frac{\partial v_z}{\partial x}+v_y\frac{\partial v_z}{\partial y}+v_z\frac{\partial v_z}{\partial z}$$

and the continuity equation:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0 \quad (6)$$

where ρ - specific mass, υ – kinematic viscosity coefficient.

Initial conditions are known and they are: p, v_x, v_y, v_z, at time t = 0 in the whole range of liquids in motion. Boundary conditions consist in knowing the velocity vector (v) or the pressure (p). It should be noted that the specific mass of the fluid depends on the concentration of the pollutant. Fluid movement can take place under laminar or turbulent motion. Under turbulent conditions the fluid particle through a very irregular trajectory varying over time. Current velocity at a same point is the random character limits.

The physicist Reynolds, who demonstrated turbulent flow regime and proposed that the system of Navier-Stokes equations are adding additional terms. Thus the idea was to decompose each size of turbulent fluid motion into two components: a mediated temporal component (details ignores turbulent motion) and a pulsating component (turbulence movement is just details). Due to turbulence appear the consistent efforts tensors, called the uniform friction tensors of the apparent effort. Reynolds equation can be written as:

$$X-\frac{1}{\rho}\frac{\partial p_{xx}}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}+\frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z}=\overline{\frac{dv_x}{dt}} \quad (7)$$

Efforts resulting from the sum of the consistent efforts due the viscosity, whit efforts of apparent friction, explained on a normal plane to the axis ox:

$$p_{xx}=-\bar{p}-\overline{pv^2{}_x};\tau_{xy}=\mu\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)-\overline{pv'_x v'_y} \quad (8)$$

Juxtaposition of equations (5) and (8), Navier-Stokes equations for turbulent motion remain unchanged and consistent efforts include both terminally viscosity and velocity pulsations:

$$X-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\upsilon\Delta\overline{v_x}-\left(\frac{\partial\overline{v'^2_x}}{\partial x}+\frac{\partial\overline{v_x'v_y'}}{\partial y}+\frac{\partial\overline{v_x'v_z'}}{\partial z}\right)=$$

$$=\frac{1}{\rho}\left(\frac{\partial p'_{xx}}{\partial x}+\frac{\partial \tau'_{yx}}{\partial y}+\frac{\partial \tau'_{zx}}{\partial z}\right)$$

$$Y-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial y}+\upsilon\Delta\overline{v_y}-(\frac{\partial \overline{v'^2_y}}{\partial y}+\frac{\partial \overline{v_y' v_x'}}{\partial x}+\frac{\partial \overline{v_y' v_z'}}{\partial z})=$$
$$=\frac{1}{\rho}(\frac{\partial p'_{yy}}{\partial y}+\frac{\partial \tau'_{zy}}{\partial z}+\frac{\partial \tau'_{xy}}{\partial x})$$

$$Z-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial z}+\upsilon\Delta\overline{v_z}-(\frac{\partial \overline{v'^2_z}}{\partial z}+\frac{\partial \overline{v_z' v_x'}}{\partial x}+\frac{\partial \overline{v_z' v_y'}}{\partial y})=$$
$$=\frac{1}{\rho}(\frac{\partial p'_{zz}}{\partial z}+\frac{\partial \tau'_{xz}}{\partial x}+\frac{\partial \tau'_{yz}}{\partial y}) \quad (9)$$

Near the solid walls the high weight have the tangential equations because of the viscosity, while inside the fluid the high percentage have the tangential friction apparent effort. In canals, water flow is characterized to be generally in non-permanent movement, it mean that the current velocity in any point of the space occupied by the fluid depends in time. For a sewerage network, the layers differ, but the modeling is obtained in the same manner as if the movement gradually varied for a homogeneous liquid. These unknowns are determined by solving the Saint-Venant equations represented by:

$$B\frac{\partial z}{\partial t}+\frac{\partial Q}{\partial x}=0,$$

continuity equation (10)

$$\frac{\partial z}{\partial x}+\frac{1}{g\omega}\frac{\partial Q}{\partial t}+\frac{2Q}{g\omega^2}\frac{\partial Q}{\partial x}-\frac{Q^2}{g\omega^3}\frac{\partial \omega}{\partial x}+\frac{Q^2}{K^2}=0 \quad (11)$$

dynamics equation, where:

Q - flow; z - free surface share, B - width,
ω - section of flow, K - module of flow. These equations must be solved simultaneously with the dispersion equation for gradually varied fluid movement:

$$\frac{\partial(\omega\overline{c})}{\partial t}+\frac{\partial(\omega\overline{uc})}{\partial x}=\frac{\partial}{\partial x}(\omega D_t\frac{\partial \omega\overline{c}}{\partial x})+q \quad (12)$$

, where replacement is Q= $\omega\overline{c}$.

To solved it used the method of finite differences and the theoretical calculations and show that the scheme is rapidly convergent calculation, meeting the requirements of consistency and stability. Consistency condition occurs when the finite difference equation obtained tends to limit the differential equation and calculation steps (Dt, Dx) which tend to zero. The condition of stability of numerical solution of the diffusion equation $\frac{\partial c}{\partial t}=Dt\frac{\partial^2 c}{\partial x^2}$, with initial and boundary condition data, believes that at a time t> 0, the numerical solution is stable. You must know the turbulent dispersion coefficient, Dt and pollutant source term q (positive or negative), depending on the nature of the pollutant. The term who express the dispersion to convection of the uniform or gradually varied movement of fluid substitute the diffusion in the normal plan on the ox axis and the axis oy, oz. This shows that diffusion is three-dimensional and the motion is one-dimensional of fluid. Diffusion along the flow axis is negligible compared to convection and diffusion and will be analyzed only in the normal plane to the flow axis (ox). Dispersion coefficient, in one-dimensional motion, including the influence of geometry section and velocity distribution in current section can be determined either by field experimental method or by calculation methods proposed by some researchers. (eg calculation method Bens). Saint-Venant equations are typical for SWMM program (Storm Water Management Model) - for transport mode that calculates pollutant loading from a sewerage network. Flows are known and the pollution load are entering in points from the sewer system.

RESULTS AND DISCUSSIONS

To determine the evolution of pollutant can achieved mathematical program like SMS (Surface-water Modeling System-SMS) RMA4. Type are resolved convection processes – diffusion and can be used in analysis of the evolution of any conservative pollutants (suspension or dissolved in water). The software package is effective for managing the entire process of modeling surface water: from importing topographic and hydrodynamic data up to visualize and analyze solutions. RMA4 uses hydrodynamics resulting from RMA2 and calculates a solution of dispersion equation using the finite element method. Influence of convective turbulence field is reflected by using turbulent diffusion coefficients in the directions x and y. Turbulence effects are taken into account by using turbulent viscosity, which is also a means of ensuring the numerical stability of the solution. Another software package is model-USEPA SWMM (Storm Water Management Model). Fundamental basic algorithm is as follows:

- at some point in time we know all values and quality hydraulic system;
- pollutant insert a node after a period in which it is spread only variable element concentrations seeks changes in nodes and route;
- is established at a time of increased pollutant concentrations at the point of measure.

To identify the source of the pollutant is necessary to identify the coordinates of nodes and drainage network characteristics (flow, velocity, diameter, length, type section) and the amount of pollutants in different areas of injection. Following mathematical modeling to obtain hydraulic data and water quality in sewer. The results are determined by calculating the hydraulic flow network section using Manning's equation and qualitative results provide information on concentrations of pollutants along the network and the intersection (nodes).

CONCLUSIONS

Mathematical modeling of transport processes and dispersion of pollutants in water developed along with great progress made in the field of computers.

To develop the mathematical model of water flow is RMA4 program. It uses the system of Navier-Stokes equations as Reynolds after x and y cartesian coordinates with the equation of continuity for incompressible fluids with free surface in turbulent motion.

Modeling pollutants can present in SWMM calculation program, using Saint-Venant equations and the movement is gradually varied dimensional for the stratified currents in pipes and channels.

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