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COMPARATIVE ANALYSIS OF THE RESULTS OBTAINED FROM SOLVING SMALL SPHERICAL TRIANGLES THROUGH LEGENDRE METHOD, AND THROUGHSOLDNER METHOD

Florita-Ionela STRUGARI[1], Raluca-Camelia MURESAN[1], Alexandra-Madalina GRAD[1], Alina GIURGIU[1]

Scientific Coordinators: Prof. PhD Eng.Mircea ORTELECAN[1], Assist. PhD Eng. Tudor SĂLĂGEAN[1]

[1]University of Agricultural Sciences and Veterinary Medicine of Cluj-Napoca, CaleaMănăştur 3-5, 400372, Cluj-Napoca, Romania | Tel: +40-264-596.384 | Fax: +40-264-593.792 Email: florita_strugari@yahoo.com

Corresponding author email: florita_strugari@yahoo.com

Abstract

The observation executed on the topographical surface in the points of the triangulation network are projected on the reference surface which is the ellipsoid of rotation.The ellipsoid of rotation can be approximated with a sphere of avarageradius.The triangulation networks of first order are always solved on the reference ellipsoid or on the sphere of medium radius.Legendre and Soldner created two methods of solving the small spherical triangles using relations from the plane trigonometry to make easier the calculation.The comparison of the obtained results through this two methods was realized for triangulation points form Cluj county.

***Key words**: ellipsoid, Legendre, Soldner, triangles spherical.*

INTRODUCTION

The geodesic triangulations are formed by ellipsoidal triangles, so they are defined on the surface of the reference ellipsoid.
The side lengths of the triangles vary between $30 \div 50$ kilometres and rarely get to 60kilometres in the case of the first order geodesic networks.
Because on relatively small surfaces, the reference ellipsoid can be approximated with a sphere of medium radius, solving the ellipsoidal geodesic triangles can be reduced to solving some spherical triangles. (Ortelecan, 2006).
Solving this problem means calculating the sides length of the triangles from the triangulation network starting from a known side and having all the angles of the triangle determined.
When solving the spherical geodesic triangles it is not recommended to use the known formulas of the spherical trigonometry, because the spherical excess (the plusses over 200^g) have small values.(Ghiţău, 1983).

MATERIALS AND METHODS

The Optical Micrometer Theodolite Theo 010 (Figure 1) is suited for all geodetic work. The principal application of the instrument was for triangulations of 2^{nd} to 4^{th} order, precision traversing (above and below ground).
The instrument is compact and completely self-contained. The accuracy of Theo 010 is 2". (M. Neamţu et al., 1982).
The horizontal and vertical scale intervals of the micrometer is 1". The altitude bubble sensitivity is 20"/2mm. The circles have double line graduations, the actual division being between the two lines. By turning the micrometer head, the circles apparently move in opposite directions until the upper and lower double lines are made to coincide.
For this purpose, adequate special methods are going to be used, namely:

- Soldner method (the method of the additaments);
- Legendre method (the method of the developments in series).

Figure 1 - Theo 010

Legendre Method (Developments In Series)

The Legendre method is one of the most used methods when solving the small geodesic triangles. The method is based on the usage the following considerations:
Considering the sides of the spherical triangle equal with the sides of the plane triangle , the angles of the spherical triangle must be offset with 1/3 of the spherical excess. So the small spherical triangles are going to be solved by applying the plane trigonometry formulas.(Dima, 2005)
The steps of solving the small spherical triangles are :
1. Calculation of the spherical excess
2. Offseting the angles of the spherical triangle
3. Calculation of the angles of the plane triangle by correcting the angles with 1/3 of the spherical excess
4. Calculation of the sides by applying the sinus theorem from the plane triangle

Soldner Method(Aditamentelor)

Aditamentelor method. With Aditamentelor method, which was introduced by J.G. Soldner in 1810, any small ellipsoidal triangle (the triangle located on the average Gauss sphere) is solved by a plane triangle, whose angles are equal to those of the ellipsoidal triangle, but the triangle side will change with calculable values, which are called aditamente.(Dima, 2005)
Calculation algorithm :

· The calculation of spherical excess;

· The empirical processing of the angles in the parallel spherical triangle

· The calculation of the first side in the intermediary plan triangle: $a' = a - A_a$, whereAais called aditament for the a side and it is determined with the equation $A_a = \frac{a^3}{6R^2}$.

RESULTS AND DISCUSSIONS

Legendre Method
To solve the small triangle ellipsoid using the Legendre we need as known elements the angles 40 , 48, 219 and side “40” measured (figure 2) and reduced on ellipsoid surface and we have to determine other sides 48, 218 of the small ellipsoid triangle (Table 1).

Table 1. Coordinates of the geodetic triangulation network on the administrative territory of Cluj-Napoca

Designation point	Point no.	X	Y	Z
Dl Popesti	40	592373.84	389535.84	682.40
Dl Soporului	48	584577.58	397156.93	459.07
La Bazin	219	584181.45	391671.64	485.98

Calculation algorithm:
Excess spherical ε^{cc}
$$\varepsilon^{cc} = (\alpha) + (\beta) + (\gamma) - 200^g$$

- calculation of excess spherical ε^{cc} for small elipsodic triangle . Of which tips are denoted 40, 48, 219.
- processing empirical angle in spherical triangle coresondent applying the principle of equal influence of measurement errors on the three angles:

$$40^\circ + 48^\circ + 219^\circ = 200^g + \varepsilon^{cc} + \omega .$$

Where with ω noted the total error of measurement of the three angles.

$$\omega = 40^\circ + 48^\circ + 219^\circ - (200^g + \varepsilon^{cc}) .$$

Based on this principle, the three angular corrections will be equal to each other

$$v'_{40} = v'_{48} = v'_{219} = -\frac{\omega}{3}$$

so that they can obtain preliminary processed angles spherical triangle

$$40 = 48^{\circ} - \frac{\omega}{3};$$

$$48 = 48^{\circ} - \frac{\omega}{3};$$

$$219 = 219^{\circ} - \frac{\omega}{3}.$$

The sum of these angles will be:

$$40 + 48 + 219 = 40^{\circ} + 48^{\circ} + 219^{\circ} - \omega = 200^{g} + \varepsilon$$
.

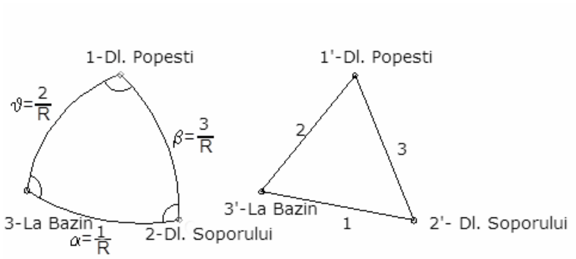

Figure 2 - Solving small ellipsoidal triangles through Legendre theorem

- angles calculation (processed provisional) in the inermediate plan triangle (auxiliary) in accordance with the theorema Legendre:

$$40' = 40 - \frac{\varepsilon^{cc}}{3};$$

$$48' = 48 - \frac{\varepsilon^{cc}}{3};$$

$$219' = 219 - \frac{\varepsilon^{cc}}{3}.$$

Sum of the angles in the triangle interimediate plan will be :

$$40' + 48' + 219' = 40 + 48 + 219 - \varepsilon^{cc} = 200^{g}.$$

- intermediate triangle solving plan in which the sum of the angles fulfills the condition can be achieved by known theorem from the plane trigonometry:

$$\frac{40}{\sin 40'} = \frac{48}{\sin 48'} = \frac{219}{\sin 219'}$$

If the magnitude established is noted:

$$modul = \frac{40}{\sin 40'}.$$

results sides 48 and 219, identical in triangle plan and in small ellipsoid triangle:

48 = modul sin 48';

219 = modul sin 219'.(Table 2)

Table 2. Calculation of the triangle sights lengths of small ellipsoid triangle with Legendre method

Triangle peak	Angle measured	Corection -ω/3	Offset angle in the small ellipsoidal triangle	Corection -ε/3	Offset angle in the plane triangle	Sine of the angle in the plane triangle	Module	Length of the edge in the plane triangle and in the small ellipsoidal triangle	Name of the edge
40	33.04158	0.118927712	33.16050771	-0.119381045	33.04112667	0.496019715		5499.575016	48-219
48	55.31308	0.118927712	55.43200771	-0.119381045	55.31262667	0.763586047		8466.193217	40-219
219	111.6467	0.118927712	111.7656277	-0.119381045	111.6462457	0.98331331	11087.4122	10902.39995	40-48
				[]	200				

Soldner Method

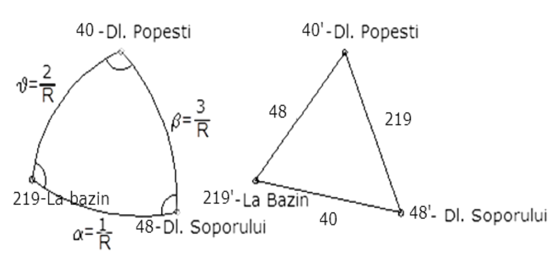

Figure 3 - Solving small ellipsoidal triangles with theorem Soldner

Solving triangle intermediate plan in which are known: side a and all three angles $40' \equiv 40$; $48' \equiv 48$; $219' \equiv 219$, also by using known theorems of plane trigonometry:

$$\frac{40'}{\sin 40} = \frac{48'}{\sin 48} = \frac{219'}{\sin 219}.$$

Is denoted like the Legendre method:

$$Modul = \frac{40'}{\sin 40},$$

48' = Module sin 48;

219' = Module sin 219 (Figure 3)

Calculation side 48 and 219 in the little ellipsoid triangle:

$$48 = 48' + 1_2,$$

$$219 = 219' + 1_3;$$

where:

$$1_{48} = \frac{(2')^3}{6R^2}.$$

$$1_{219} = \frac{(3')^3}{6R^2};$$

As shown, in calculating aditamentelor 1_{48} and 1_{219} were used sides 48' and 219' from intermediar plan triangle not the sides 48 and 219 from small ellipsoidal triangle,(Table 3) approximation is allowed to sides less than 100 km.

It can be shown that through calculation of aditamentelor in the case of small ellipsoidal triangles (s < 60 km) is enough to the mean for total curve $K = \frac{1}{R^2}$, for territories covering a large area with 5° latitude north and south respectively (about 1000 km in total on the north - south). For our country that meets the condition referred and where 48_{medium} = 46°, it can be used:

$$R = R_{46^\circ} \approx 6378957 \text{ m},$$

which $\frac{1}{R^2} \approx 4.0959 \cdot 10^{-15}$.

Table 3. Calculation of small ellipsoidal triangle sides lengths with Legendre method

Name side	**Value side**
48-219	5499.575016
40-219	
40-48	
(48-219)'=	5499.574335
Module=	11087.27257
(40-219'=	8466.137613
(40-48)'=	10902.24833
40-219	8466.140098
40-48	10902.25364
Name side	**Value side**
48-219	5499.575016
40-219	8466.140098
40-48	10902.25364

CONCLUSIONS

It can be seen that the results obtained through the Legendre's method and Soldner's method are approximately equal, the differences are insignificant and it can be showed the results of the coordinates of in the next table (Table 4).

Table 4. Side differences obtained from the two methods of calculation

Methods	Side lenght in the plan triangle and little ellipsoid triangle [m]	Side name	Differences between methods
Legendre method	5499.575016	48-219	0
	8466.193217	40-219	0.053118725
	10902.39995	40-48	0.146309453
Soldner method	5499.575016	48-219	
	8466.140098	40-219	
	10902.25364	40-48	

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