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# COMPARATIVE STUDY OF BASIC GEODETIC PROBLEMS SOLVED ON THE ELIPSOID AND SPHERE

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***Abstract***

*This paper aims to present a comparative study of basic geodetic problems solved on the ellipsoid or sphere. The basic geodesic problem refers to the geographic coordinate system that consists of ellipsoidal: geodesic problem direct and inverse geodesic problems. Thus, using direct geodetic problem is calculated coordinates of geodesic points forming a network of the geodesic and geodesic inverse problem element is calculated initial surveying , those distances and azimuth , and verify the calculations made at the geodesic problem directly. The ultimate goal of the calculations performed on the reference ellipsoid is the determination the coordinates of the points of the geodetic networks of support. There are several ways of solving the basic geodetic and this diversity was conditioned by the need to decrease the volume, increase the accuracy of the final results, even in terms of geodetic distances, as well as by means of calculation under consideration at your disposal.*

***Key words:*** *Ellipsoid, Inverse problem, Direct problem, Sphere, Geodetic problems solved*

## INTRODUCTION

Geodetic measurements are effectuated on the physical surface of the Earth which is a complex and not at all plain surface (Dima, 2005).

For obtaining unitary coordinates of points belonging to the geodetic network support, these measurements must be related to a unique surface.

After some extensive research it had been established that this surface may be taken as the quiet surface of the oceans and seas imaginary prolonged beneath the continents, and the solid that is made out of this figure is called geoid. The surface that approximates best the geoid, is the rotation ellipse with low flatting at poles.

The rotation ellipse is a mathematical figure generated by spinning a meridian ellipse around the pole axis (Figure 1).

For small surfaces, the rotation ellipse may be assimilated with the sphere of medium radius, which presents facilities in mathematical operation effectuation.

Solving the first order triangulation networks is performed on the reference surface. Following the triangulation network points to be given by geodetic coordinates B, L or geographic coordinates $\varphi$ and $\lambda$.

Geodetic latitude and longitude coordinates are commonly measured in degrees, minutes, and sexagesimal seconds (Postelnicu, 1980).

The ellipsoidal altitude (HE) or elevation has two components: orthometric altitude, denoted by HORT, which represents the height of point measured by leveling the surface from the geoid and geoid undulation, denoted by N, which represents the height of the geoid from the ellipsoid surface (http://www.scrigroup.com/geografie/geologie/suprafete-de-referinta21138.php).

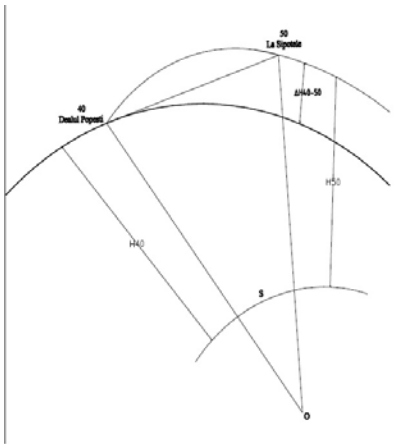

Figure 1. Projection on the ellipsoid

## MATERIALS AND METHODS

The state geodetic network consists of high-order triangulation points (I, II, III) and low-order triangulation points (IV, V). This network is presented as a compact network of triangles combined with quadrilateral shapes with both diagonals observed, having scientific and practical purpose (Ortelecan, 2006).

The first order network has points arranged in triangle shape, equilateral if possible, assuring a medium sides length of 25 – 30 km (Manualul inginerului geodez, 1973).

Azimuth and zenith observations for triangulation networks of I and II order are performed with high precision theodolites which have reading accuracy of 0.3 and 0.5 seconds, and the measurement method used is the binary series method ( The Schreiber method). Generally for measuring first order network angles, there are used Wild T4 and Wild T3 (Figure 3). In networks of II, IV and V order, there are used Wild T2, Zeiss Theo 010A and B with a precision of 0.2 seconds (Figure 2).


Figure 2. Theo 010A and B


Figure 3. Theodolit Wild T2
(http://google imagine.com)

## RESULTS AND DISCUSSIONS

The final purpose of the calculations effectuated on the reference ellipsoid is the determination of the geodetic coordinates; latitude B and longitude L, of points belonging to the geodetic networks support. The rigorous processing operations of astronomical-geodetic determinations claims the calculation of geodetic coordinates in several phases: the calculation of temporary coordinates, necessary in the preliminary stage of rigorous processing and final coordinates calculation, after completion of proper compensation. Therefore it can be appreciated that this type of calculations hold an importantly distinguished volume, reason for they are recognized in the specialized literature as basic geodetic problem solving(http://www.acpi.ro/pages/wiki.php?lang=ra&pnu=transformariCoordonate).

The first basic geodetic problem, known as direct geodetic problem, consists of determining geodetic coordinates B2, L2 of point S2 and geodetic azimuth A2, known as inverse geodetic azimuth, according to B1, L1 coordinates of point S1, geodetic azimuth A1, known as direct geodetic azimuth, and length of geodetic line s between points S1, S2 (Figure 4).

The succesive use of direct geodetic problem is known as coordinates transit (Figure 5).

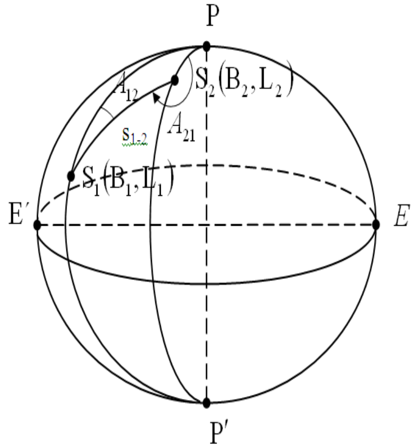


Figure 4. Representation on the ellipsoid

Known data :

B1 = $46^0$ 49' 19'',5088

L1 = $23^0$ 33' 07'',50596

A1 = $46^0$ 38' 41'',8937

s = 13549.88851 m

Required:

B2= ?

L2= ?

A'1= ?

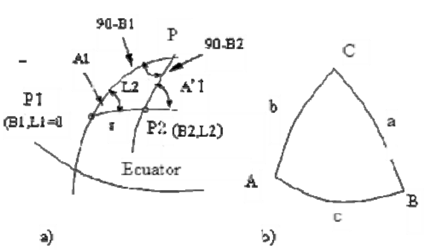


Figure 5.Direct geodetic problem

Calculation steps :
1.Transformation of sexagesimal minutes and seconds in decimal degrees;
2. Transforming the geodetic length in radians;
3. Using spherical trigonometry relations of Delambre;

$$\sin\frac{a}{2}\cos\frac{B-C}{2}=\sin\frac{b+c}{2}\sin\frac{A}{2}$$

$$sin\frac{a}{2}sin\frac{B-C}{2}=sin\frac{b-c}{2}cos\frac{A}{2}$$

$$cos\frac{a}{2}cos\frac{B+C}{2}=cos\frac{b+c}{2}sin\frac{A}{2}$$

$$\cos\frac{a}{2}\sin\frac{B+C}{2}=\cos\frac{b-c}{2}\cos\frac{A}{2}$$

4.Adaptation of angles from trigonometric relations with tohse from the spheric triangle from the example before us (Table 1);

Table 1. Elements of the triangle

| | |
|---|---|
| $A = A_1$ | $a = 90^0 - B_2$ |
| $B = 180^0 - A'_1$ | $b = 90^0 - B_1$ |
| $C = L_2$ | $c = s'$ |

$$\frac{B-C}{2}=\frac{180^0-A_1^{'}-L_2}{2}=90^0-\frac{A_1^{'}+L_2}{2}$$

$$\frac{b-c}{2}=\frac{90^0-B_1-s'}{2}$$

$$\frac{B+C}{2}=\frac{180^0-A_1^{'}+L_2}{2}=90^0-\frac{A_1^{'}-L_2}{2}$$

$$\frac{b+c}{2}=\frac{90^0-B_1+s'}{2}$$

5.Coordinates calculation of point S2 with Delambre relations;

I

$$\sin\frac{90°-B_2}{2}\sin\frac{A_1^{'}+L_2}{2}=\sin\frac{90°-B_1+s'}{2}\sin\frac{A_1}{2}$$

II

$$\sin\frac{90°-B_2}{2}\cos\frac{A_1^{'}+L_2}{2}=\sin\frac{90°-B_1-s'}{2}\cos\frac{A_1}{2}$$

III

$$\cos\frac{90°-B_2}{2}\sin\frac{A_1^{'}-L_2}{2}=\cos\frac{90°-B_1+s'}{2}\sin\frac{A_1}{2}$$

IV

$$\cos\frac{90°-B_2}{2}\cos\frac{A_1^{'}-B_2}{2}=\cos\frac{90°-B_1-s'}{2}\cos\frac{A_1}{2}$$

By dividing relations I and III to II and IV results :

$$tg\frac{A_1^{'}+L}{2}=\frac{\sin\frac{90°-B_1+s'}{2}}{\sin\frac{90°-B_1-s'}{2}}tg\frac{A_1}{2}=$$

$=0,433453699$

$$tg\frac{A_1^{'}-L}{2}=\frac{\cos\frac{90°-B_1+s'}{2}}{\cos\frac{90°-B_1-s'}{2}}tg\frac{A_1}{2}=$$

$=0,430771149$

$A_1^{'}+L$ =2arctg 0,433453699 =
=46,86899742

$A_1^{'}-L$ =2arctg 0,430771149 =
=46,60996646

$A_1^{'}$ =46$^0$.73948194=46$^0$ 44’ 22”,13498
$L$ =0$^0$.129515482= 0$^0$ 07’ 46”,25573
$L_2$= $L_1$+L=23.55208+0.129515482=
=23$^0$ 40’ 53”,76169

$$tg\frac{90-B_2}{2}=\frac{\sin\frac{A_1^{'}-L}{2}}{\sin\frac{A_1^{'}+L}{2}}tg\frac{90-B_1+s'}{2}=$$

$=0,394862831$

$$tg\frac{90-B_2}{2}=\frac{\cos\frac{A_1^{'}-L}{2}}{\cos\frac{A_1^{'}+L}{2}}tg\frac{90-B_1-s'}{2}=$$

$=0,394862831$

90-$B_2$=2arctan 0,394862831 =43,09444

$B_2$= 90-43.09444=46.90555779=
=46$^0$ 54’ 20”,00804

The second basic geodetic problem known as inverse geodetic problem consists of determining the length of geodetic line s and geodetic inverse and direct azimuth A1 and A2, when the geodetic coorindates of points S1 and S2 are known (Figure 6).

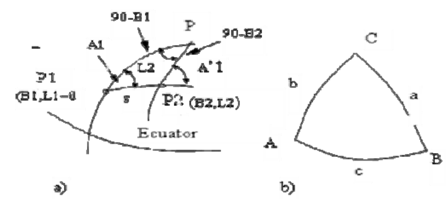


Figure 6. Inverse geodetic problem

Known data :

B1 = 46$^0$ 49’ 19”,5088

L1 = 23$^0$ 33’ 07’’,50596

B2 = 46$^0$ 44’ 19”,00879

L2 = 23$^0$ 40’ 53’’,03729

Se cere:

A1 = ?

A’1 = ?

s = ?

Calculation steps:
1. Transformation of sexagesimal minutes and seconds in decimal degrees (Table 2);
2. Adapting the angle values from trigonometric sexagesimal relations with those from the spheric triangle from the example before us;
3. The use of Delambre relations from spherical trigonometry;
4. Calculating the direct azimuth, inverse azimuth and the geodetic line (Table 3).

1.

Table 2. Measured elements

| *Coord.geod.* | *Grade [ º ]* | *Minute [ ‘ ]* | *Secunde [ “ ]* | *Grade decimale* |
|---|---|---|---|---|
| *B1* | *46* | *49* | *19,5088* | *46,82208578* |
| *L1* | *23* | *33* | *07,52596* | *23,55208499* |
| *B2* | *46* | *44* | *19.00879* | *46,73861355* |
| *L2* | *23* | *40* | *53.03729* | *23,68139925* |
| *L= L2 - L1* | 0 | 7 | 45,53133 | *0,129314258* |

2.

Table 3. Elements of the triangle

| | |
|---|---|
| A = L2 | a = s' |
| B = $180^0$-A'$_1$ | b = $90^0$ – B$_1$ |
| C = A1 | c = $90^0$ – B$_2$ |

$$\frac{B-C}{2}=\frac{180^0-A_1^{'}-L_2}{2}=90^0-\frac{A_1^{'}+L_2}{2}$$

$$\frac{b-c}{2}=\frac{90^0-B_1-s'}{2}$$

3.

$$\frac{B+C}{2}=\frac{180^0-A_1^{'}+L_2}{2}=90^0-\frac{A_1^{'}-L_2}{2}$$

$$\frac{b+c}{2}=\frac{90^0-B_1+s'}{2}$$

I

$$\sin\frac{s'}{2}\sin\frac{A_1^{'}+A_1}{2}=\cos\frac{B_2+B_1}{2}\sin\frac{L}{2}$$

II $$\sin\frac{s'}{2}\cos\frac{A_1^{'}+A_1}{2}=\sin\frac{B_2-B_1}{2}\cos\frac{L}{2}$$

III $$\cos\frac{s'}{2}\sin\frac{A_1^{'}-A_1}{2}=\sin\frac{B_2+B_1}{2}\sin\frac{L}{2}$$

IV $$\cos\frac{s'}{2}\cos\frac{A_1^{'}-A_1}{2}=\cos\frac{B_2-B_1}{2}\cos\frac{L}{2}$$

4.

$$tg\frac{A_1^{'}+A_1}{2}=\frac{\cos\frac{B_2+B_1}{2}}{\sin\frac{B_2-B_1}{2}}tg\frac{L}{2}=1{,}060880686$$

$$tg\frac{A_1^{'}-A_1}{2}=\frac{\sin\frac{B_2+B_1}{2}}{\cos\frac{B_2-B_1}{2}}tg\frac{L}{2}=0{,}000822362$$

$A_1^{'}+A_1$ =2arctg1,060880686= =93.38417667

$A_1^{'}-A_1$ =2arctg 0,000822362 = =0,094235716

By solving the sistem the result is:

$A_1^{'}$= (93.38417667+0,094235716)/2 = $46^0$.7392062= $46^0$ 44' 21",14230277

$A_1$ = (93.38417667- 0,094235716)/2 =$46^0$.64497048= $46^0$ 38' 41",89372535

For calculating the length of the geodetic line, relation I is divided by relation III, and for verification, divide relation II by IV.

$$tg\frac{s'}{2}=\frac{\sin\frac{A_1^{'}-A_1}{2}}{\sin\frac{A_1^{'}+A_1}{2}}ctg\frac{B_2+B_1}{2}=0{,}001061981$$

s' = 2arctg0,001061981= 0,00212396

s = s'R=13549,88851 m

$$tg\frac{s'}{2}=\frac{\cos\frac{A_1^{'}-A_1}{2}}{\cos\frac{A_1^{'}+A_1}{2}}tg\frac{B_2-B_1}{2}=0{,}001061981$$

s' = 2arctg0,001061981= 0,00212396

s = s'R=13549,88851 m

There are known several procedures of solving basic geodetic problems. This diversity was conditioned by continuos necessity of math movement volume, of increased accuracy for the final results, even in large geodetic distances, such as available means of calculation.

Nowadays some of the procedures, know a reduce applicability, for example the ones that are based on logarithm usage, such that would not be used in the present paper.

There are several classification criteria of the methods and procedures of geodetic coordinates calculation on the reference ellipsoid, according to the principal element within these calculations. One of the current classification criteria used, accepted within this paper, regards as principal element the length of the geodetic line *s*. From this point of view it can be distinguished: solving methods for short geodetic distances ( s<60 km ), medium distances ( 60< s < 600 km ), and long distances ( s>600 km ). Within these three methods there will be presented those solving procedures which have historical priority or know a current applicability in the geodetic use.

Because the necessary formulas used for solving inverse geodetic problems are acquired from the formulas used for solving direct geodetic problem, by applying suitable mathematical transformations (for example developmental reversals in sequence), will present further the deduction principles of formulas just for the direct geodetic problem, nevertheless numeric formulas and examples will be presented for solving both basic geodetic problems.

Among the areas where are involved solutions of basic geodetic problems we can mention: achievement of support geodetic network by the first order (where appears short geodetic distances, on an average of 30-40 km), of continental and intercontinental geodetic network(medium geodetic distances), air navigation problems, missile technic,(long geodetic distances). Basically the problem begins with the calculation of the distance and geodetic azimuth between the points of knowing coordinates, placed at large distances, tens of thousands of kilometers, such that the application in solving basic geodetic problems are nelimitated. Another problem that needs to be solved is the calculation precision of geodetic coordinates, distinguishing exactly and approximately methods. As the geodetic distances grow, the exactity in calculations has special meanings. As well as other geodetic calculation, the basic geodetic problem follows that the calculation errors to be 10 times smaller than medium errors wich characterized the field opperations. Thereby, can be showed that in gedetic triangulation of I order is necessary that the approximation of calculation for the geodetic coordinates B and L to be ±0'',0001, also for geodetic azimuths A, $\pm 0^{cc}$,001, and for geodetic distances s, ±0,001 m. These type of aproximations will be exagerated in the moment when geodetic distances are at the point of hundreds of kilometers, that were determined with radar apparatus, in this case the fields errors are higher. As soon as the laser technic of distances determination will be improved and also the processing methods(in special precise deduction on ellipsoid surface, based on knowing geoid profile) solving basic geodetic problems with a higher precision when are met medium and large geodetic distances. (Ghiţău et al., 1997)

## CONCLUSIONS

Determining the minimal distance between two points of the ellipse, as a mathematical figure of the Earth, it presumes a very laborious calculation. The results obtained by this method are more accurate, so more closer to the true value. Also, modern navigation systems ( GPS, DGPS ) use as a mathematical form of the earth, the ellipse (WGS 84).

While calculations on the sphere are more simplified that those on the ellipse.

For example for executing small-scaled maps, the globe may be assimilated with a sphere, because if it will be considered a sphere with a equatorial radius of 15 cm, the polar radius will be 0.5 mm smaller than the equatorial one, therefore negligible. For this reason, in practical cartography problems, the globe will be considered a sphere. For calculating length of the geodetic line smaller than 60 km, the formulas assure precision of 1 mm for length s and 0'',001 for azimuth A. Calculation of geodetic-geographic coordinates on the ellipse represent a basic geodetic problem of triangulation and trilateration of first orded. For the other orders, calculations are performed on the projection planes.

The direct and inverse geodetic problem serve for the mutual control, the results must coincide in the approximation of: 0'',0001 for geodetic coordinates, and 0'',001, for azimuth's A, and 0.001 m for length's s.

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