Journal of Young Scientist, Volume III, 2015
ISSN 2344 - 1283; ISSN CD-ROM 2344 - 1291; ISSN Online 2344 - 1305; ISSN-L 2344 – 1283

COMPARATIVE ANALYSIS REGARDING THE ACCURACY DETERMINATION OF A POINT, USING THE LINEAR INTERSECTION METHOD AND MULTIPLE COMBINED INTERSECTIONS METHOD

Rares Catalin OROS, Diana-Ioana PLOSCARIU, Razvan Casian REBREAN, Vasile Beniamin TOLOMEIU

Scientific Coordinators: Prof. PhD Eng. Mircea ORTELECAN, Lect. PhD Eng. Tudor SĂLĂGEAN

University of Agricultural Sciences and Veterinary Medicine of Cluj-Napoca, Calea Mănăştur 3-5, 400372, Cluj-Napoca, Romania | Tel: +40-264-596.384 | Fax: +40-264-593.792 Email: diana_0410_lavinia@yahoo.com

Corresponding author email: diana_0410_lavinia@yahoo.com

Abstract

This paper aims to present a comparative analysis regarding the accuracy determination of a point, using the linear intersection method and multiple combined intersections method. To determine the coordinates of the new points we always measure a much higher number of directions than we need.Those directions helps to discover measurements errors, a better precision.Before definitive calculations of network points, there will be made some compensations with the purpose to determine each point one time with a higher probability of it's coordinates value.Higher order triangulation networks are compensated by rigorous methods namely:indirect measurements method and conditionated measurements method.Wichever method of compensation applied, before compensation is introduced in the calculation direction measurements are being checked on the field, are reduced to the center of points and are also reduced in Gauss or Stereografic projection plan.By solving the triangulation networks aimed at inducing the plane coordinates of geodetic points, using known quantities of plane coordinates geodetic points and connecting lines between geodetic points.

Keywords: *point, intersection, error, precision, compensation, network*

INTRODUCTION

Preliminary processing of geodetic observations used in triangulation networks is to determine the elements needed to build functional-stochastic model of the actual processing and reduction of network observations considerer on the same reference surface ellipsoid or projection plan). The problem of errors propagation in triangulation networks aims to highlight the influence of triangulation networks shape over the determination errors of length, orientation of directions in the network and coordinates of points (Ghiţău, 1983; Ortelecan, 2006).

Probable error propagation in a network triangulation can be determined only with a concomitant compensation, calculating the mean square error of any element compensated from the network, then there can be taken steps to improve network configuration by performing new measures and a new compensation (Ghiţău, 1997; Postelnicu and Coatu, 1980). For immediate practical purposes, the optimal configuration problem of network triangulation can be reduced to the possibility of establishing characteristic errors of a network formed by or combination of these (Moldoveanu chains of triangles, rectangles with two observed diagonals, polygons with a central point, 2002; Dima, 2005).

Wichever way of achieving the network triangulation, this requires the determination of an initial length using the older process development geodetic base either by direct measurement using electromagnetic devices. Geodetic bases measurements or of initial directions is a critical operation that must be very precise because when an closure error is detected in a triangulation series on a final direction, is necessary to ensure that this error is due to propagation angular errors and less to the errors of geodetic base measurements

(Ortelecan and Sălăgean, 2014).

MATERIALS AND METHODS

Linear intersection method is to determine the coordinates of a new point P2 (XP2,YP2), knowing only the old point coordinates 309 (X309,Y309), 300 (X300, Y300), P1 (XP1, YP1), P3 (XP3, YP3), P4 (XP4, YP4) and measured distances between the new point and the old point.

To determine the coordinates of P2 (XP2,YP2) provisional coordinates are determined in the first stage XP2,YP2, using the coordinates of two points and measured distances from these to P2.

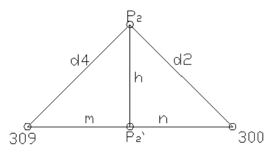

Figure 1. Linear Intersection

$$m = \frac{d_4^2 - d_2^2 + d_{309,300}^2}{2 * d_{309,300}} \qquad n = \frac{d_2^2 - d_4^2 + d_{309,300}^2}{2 * d_{309,300}}$$

$$h = \sqrt{d_4^2 - m^2}$$

Using the theory of indirect measurements will determine, in the second stage the probable corrections:

$$x_{P2} = x_{309} + m * \cos\theta_{309,300} + h * \cos(\theta_{309,300} - 100)$$

$$y_{P2} = y_{309} + m * \sin\theta_{309,300} + h * \sin(\theta_{309,300} - 100)$$

Solving the system of correction equations:

-a1dx P2 – b1dy P2 + l1 = v1
-a2dx P2 – b2dy P2 + l2= v2
-a3dx P2 – b3dy P2+ l3 = v3
-a4dx P2– b4dy P2+ l4 = v4
-a5dx P2– b5dy P2 + l5 = v5

For measured distances will write the errors equations and will obtain the following direction coefficients:

a1 =cos θ P2-P3 b1 =sin θ P2-P3
a2 = cos θ P2-300 b2 = sin θ P2-P3
a3 = cos θ P2-P1 b3 = sin θ P2-P3
a4 =cos θ P2-309 b4 =sin θ P2-P3
a5 = cos θ P2-P4 b5 = sin θ P2-P3

Calculation of free terms:

l1 = D P2-P3C- DP2-P3M
l2=D p2-300 calc- D p2-300 mas
l3=D p2-p1 calc- D p2-p1 mas
l4=D p2-309 calc- D p2-309 mas
l5=D p2-p4 calc- D p2-p4 mas

To solve the normal equations system is required a solution condition: [pvv]→minimum, and the will be calculates the normal equations coefficients:

[paa]dxP2+[pab]dyP2+[pal]=0
[pab]dxP2+[pbb]dyP2+[pbl]=0

The matrix system of correction equations is:

AX=l=v

A=coefficients matrix
X=matrix of unknowns
L=free terms matrix
V=correction matrix of measured elements

The unknown of correction equations is calculated using the equation:

$$A=\begin{pmatrix} a1 & b1 \\ a2 & b2 \\ a3 & b3 \\ a4 & b4 \\ a5 & b5 \end{pmatrix} \quad X=\begin{pmatrix} dxP2 \\ dyP2 \end{pmatrix}$$

$$l=\begin{pmatrix} l1 \\ l2 \\ l3 \\ l4 \\ l5 \end{pmatrix} \quad v=\begin{pmatrix} v1 \\ v2 \\ v3 \\ v4 \\ v5 \end{pmatrix}$$

$$p=\begin{pmatrix} p1 & 0 & 0 & 0 & 0 \\ 0 & p2 & 0 & 0 & 0 \\ 0 & 0 & p3 & 0 & 0 \\ 0 & 0 & 0 & p4 & 0 \\ 0 & 0 & 0 & 0 & p5 \end{pmatrix}$$

$$Qxy=\begin{pmatrix} Qxx & Qxy \\ Qxy & Qyy \end{pmatrix}$$

Provisional coordinates calculation:
(Δ XP2)= XP2+Δ XP2
(Δ YP2)= YP2+Δ YP2
To calculate the accuracy the following equations will be used:

$$m_0 = \pm\sqrt{\frac{[vv]}{n-k}}$$

$$m_{\Delta x_0} = \pm m_0\sqrt{Q_{11}}; m_{\Delta y_0} = \pm m_0\sqrt{Q_{22}};$$

n = the number of correction equations from the initial equation system(unsimplified);
k = the number of unknowns from the same equation system;
Q11, Q22 = wheighting coefficients established by the shown method in the theory of indirect measurements.
Multiple combined intersection is the most appropiate method, wich will be aplied whenever is possible.In this process, the stationary points will be both in the old and the new points, resulting mutual visas between points (Manualul inginerului geodez vol.II, 1973).
Multiple combined intersection
Provisional coordinates calculation will be determine from 309, 300, P1, P3, and P4, through forward intersection using the equations:
Station P1 -dz P1 + l1 = v1
-dz P1+ a2dxp2 + b2dyp2 + l2= v2
-dz P1+l3 = v3
-dz P1+l4 = v4
-dz P1+l5 = v5
Station 300
- dz 300 +a5dxP2+b5dyP2+ l5= v5
-dz 300 + l6 = v6
-dz 300 + l7 = v7
-dz 300 + l8 = v8
-dz 300 + l9 = v9

Station P4 - dz P4 + l10 = v10
-dz P4 + l11 = v11
-dz P4 + a12dxP2+b12dyP2+l12 = v12
-dz P4 + l13 = v13
-dz P4 + l13 = v13
The system contains 15 equations with 5 principal unknowns (dx,dy,dzP1,300,P4) and 13 corrections (v1,…,v13).
Applying equivalence rules of Schreiber will obtain a system of:
a2dxp2+b2dyp2+l2=v2 p=1
p=-1
a5dxP2+b5dyP2+ l5= v5 p=1
p=-1
a12dxP2+b12dyP2l12 = v12 p=1
p=-1
System solving will be in the condition of minimum [pvv]→minimum and thus the normal system of equations is represented as:
[paa]dxP2+[pab]dyP2+[pal]=0
[pab]dxP2+[pbb]dyP2+[pbl]=0
Calculation of normal equations coefficients:
Solving the normal equation system is achieved by Gauss-Dolittle method.
The most probable of coordinates of P2 :
(Δ XP2)= XP2+Δ XP2
(Δ YP2)= YP2+Δ YP2
Average error of observations :

$$m_0 = \pm\sqrt{\frac{[vv]}{n-k}}$$

Average error of probable values :

$$m_{\Delta x_0} = \pm m_0\sqrt{Q_{11}}; m_{\Delta y_0} = \pm m_0\sqrt{Q_{22}};$$

Writing matrix equation system of corrections has different accuracies and is represented by:
A*X=l=v
Calculation of elipse errors:

$$a = m0\sqrt{\lambda 1}$$

$$b = m0\sqrt{\lambda 2}$$

$$m_0 = \pm\sqrt{\frac{[vv]}{n-k}}$$

$$\lambda_1 = \frac{Q_{XX} + Q_{yy}}{2} + \frac{1}{2}\sqrt{(Q_{xx} - Q_{yy})^2 + 4Q_{xy}^2}$$

$$\lambda_2 = \frac{Q_{XX} + Q_{yy}}{2} - \frac{1}{2}\sqrt{(Q_{xx} - Q_{yy})^2 + 4Q_{xy}^2}$$

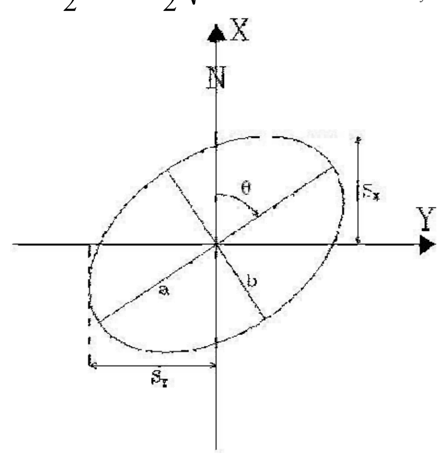

Figure 2. Elipse errors

RESULTS AND DISCUSSIONS

Indirect measurements theory applied to solve geodetic problems imposes to verify the calculation during their calculations and different stages.

These checks are performed by specified controls in the process of correction equations system and solving this system.

The final control of solving geodetic network consists in cheking related directions orientation of geodetic points.

The value of orientation for any directions can be obtain using probable coordinates of the point determined.

This method is applied for the points of II, III and IV order and has the advantage that is much economica land easier.

It is almost the same as the method used in triangulations, with the specification of that the field measurements are done faster, no matter the visibility (day-night)

In this case there are needed errors equations after the provisional coordinates are established.

Provisional coordinates are established in the same way as solving trilaterations, by conditionates measurements theory, and error equations will have the shape as in the case of distance measurements.

For the indirect measurement method the problem to find probable correction system through mathematical conditions of the network that express the related direct measurement and the data that will be calculated.

Every mathematical condition will be under the form of correction equation.The number of correction equation it is equal with the total number of effectuated measurements.Every geodetic observation has an equation and this represents a real advantage for compensation of networks.

For compensation of triangulation network through indirect observations method the unkown data for equation will be chosen from separate coordinates from station.

As a case study we performed measurements inside our campus, University of Veterinary Medicine Cluj-Napoca (Figure 3)

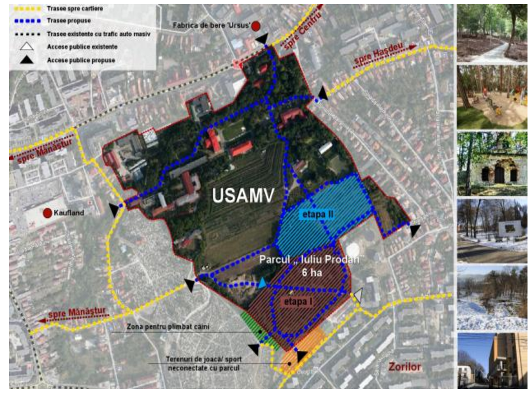

Figure 3

To determine the new point P2, we used the total station TCR805 (Figure 4).


Figure 4. Leica TCR 805

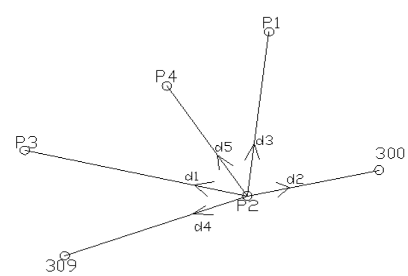

Figure 5. Linear intersection

Known coordinates

Den. Pct	x	y	z
309	585570.98	390938.3	356.123
300	585590.473	391005.6	356.85
P1	585621.914	390982	355.117
P3	585594.967	390929.7	355.566
P4	585609.607	390960.1	355.308

Provisional coordinates calculation:

m	41.345
n	28.730
m+n	70.076
h	2.130

cosω	0.998675469
ω	3.276983186
θ 309,300	78.77742215

XP_2	585584.5271
YP_2	390977.3711

Result of correction equations system:

a	b
0.213996	-0.9768344
0.206388	0.9784702
0.992367	0.1233174
0.327223	0.9449471
0.824001	-0.5665886

l_1	-0.044707
l_2	0.000000
l_3	-0.081423
l_4	0.000000
l_5	-0.081736

Results of normal equation system:

	5	0	0	0	0
	0	5	0	0	0
P=	0	0	5	0	0
	0	0	0	5	0
	0	0	0	0	5

At*P=	1.06998224	1.031939645	4.9618364	1.636115888	4.120004119
	-4.8841722	4.892351231	0.6165872	4.724735421	-2.832943003

Results for matrix of correction equation system:

A=	0.213996448	-0.9768344
	0.206387929	0.9784702
	0.992367274	0.1233174
	0.327223178	0.9449471
	0.824000824	-0.5665886

(A_t*P*A)=	9.29617815	-0.211896696
	-0.2118967	15.70382185

$(A_t*P*A)^{-1}$=	0.10760418	0.001451938
	0.00145194	0.063698358

A_t*P*L=	-0.7885977
	0.39970342

	0.002920
	0.006398
V=	0.000789
	-0.004600
	-0.001484

Calculation of standard deviation:

S_0=	0.004348477
S_{dx}=	0.001426434
S_{dy}=	0.001097492

Multiple combined intersection

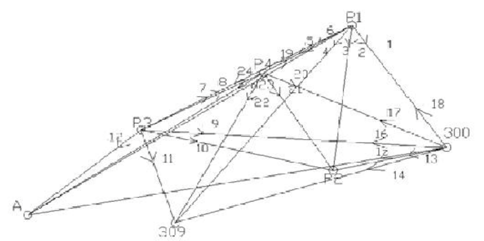

Figure 6. Multiple combined intersection

Known data:

Nr. Punct	X	Y
P1	585621,914	390982,017
300	585590,473	391005,560
P4	585609,607	390960,126
P3	585594,969	390929,716
309	585570,98	390938,25

Calculation of normal equation system:

	1	0	0	0	0	0	0
	0	-1	0	0	0	0	0
	0	0	1	0	0	0	0
P=	0	0	0	-1	0	0	0
	0	0	0	0	0	0	0
	0	0	0	0	-1	0	0
	0	0	0	0	0	1	0
	0	0	0	0	0	0	-1

A^t*P*A=	16472	-6231
	-6231	3230,7

Results of normal equation system through Gauss-Dolittle method:

	a]	b]	l]	s]	Control
	64528,879751	8935,011022	-630687,563238	-557223,672465	
	-1	-0,138465305	9,773725589	8,635260284	8,635260284
dx P2=	9,514420777	50662,374025	-179887,187871	-120289,802824	
		-1237,189027	87328,34586	77156,14581	
		49425,184998	-92558,842008	-43133,657010	
		-1	1,872706031	0,872706031	0,872706031
	dy P2=	1,872706			

Weight coefficients:

Q 11	Q22
0,000224447	0,001144346

Calculation of standar deviation:

mo=	593,3662293
S_{dxP2}=	2,364908701
S_{dyP2}=	2,669000613

Normal equation system also can be solved through matricial method:

	57,83208975	-35,1239585
	23,60985177	-14,339296
	82,13777778	-12,5054968
	33,53260736	-5,1053477
A=	59,56804764	-9,96999837
	24,31855362	-4,07023479
	78,27410904	-48,8611414
	31,9552712	-19,9474775

	-155,955297
	5,68434E-10
	-185,855618
	-6,662E-10
L=	-158,952754
	0
	-347,076853
	9,9476E-10

$(A^t*P*A)^{-1}$=	0,0002	0,000432866
	0,0004	0,001144346

$(A^T*P*A)^{-1}*A^T*P*l)$=	2,269479312
	-3,777680

If is used the Gauss method or matricial method the results will be the same.

a	b	l
21,05772253	-167,7505486	71,8592278
8,596779222	-68,48387467	0
-127,6046434	-27,91818652	623,4552676
-52,09437753	-11,39755192	-1,42109E-10
215,9892425	-45,29436696	-1818,685343
88,17723902	-18,49134788	-1,7053E-09
-118,5619252	-172,6950997	1348,059234
-48,4027033	-70,50247924	9,9476E-10

Gauss		Matricial	
dx P2=	2,269	dx P2=	2,269
dy P2=	-3,778	dy P2=	-3,778

CONCLUSIONS

Once with the appearence of electro-optic and radio telemeters, trilateration method was developed as a solving method of geodetic network and in the case of adding new points in geodetic network, the linear intersection method will be used.The necessary time elapsed on field in the case of linear intersections is shorter than the time elapsed on multiple combined intersections, and the cost of project will be reduced.

REFERENCES

Ghitau D..Geodezie si gravimetrie geodezica,Ed.Did. si Ped.,Bucuresti,1983

Moldoveanu C.,Geodezie,Ed.MatrixRom, Bucuresti,2002

Ghiţău D. et al., Teoria figurii pamantului, Editura Tehnică,Bucuresti,1997

Postelnicu Viorica and Coatu Silvia, Mica enciclopedie matematica, Editura Tehnică, Bucuresti, 1980

Ortelecan Mircea, Geodezie, Editura AcademicPres, Cluj-Napoca, 2006,

Dima Nicolae, Geodezie, Editura Univeritas, Petrosani, 2005

***Manualul inginerului geodez,vol II, Editura Tehnică, Bucureşti, 1973

Ortelecan M.,Salagean T.,Geodezie lucrari practice,Ed.Risoprint,Cluj-Napoca,2014