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ASPECTS OF SOLVING THE BASIC GEODETIC PROBLEM ON THE SPHERE

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Abstract

The purpose of this paper is to present the methods of solving the basic geodetic problems. Taking advantage of numerical integration, we solve the direct and inverse geodetic problems on the ellipsoid. In general, the solutions are composed of a strict solution for the sphere plus a correction to the ellipsoid determined by numerical integration. The problems in geodesy are usually reduced to two main cases: the direct problem, given a starting point and an initial heading, find the position after traveling a certain distance along the geodesic; and the inverse problem, given two points on the ellipsoid find the connecting geodesic and hence the shortest distance between them. Much of the early work on these problems was carried out by mathematicians—for example, Legendre, Bessel, and Gauss—who were also heavily involved in the practical aspects of surveying. If the Earth is treated as a sphere, the geodesics are great circles (all of which are closed) and the problems reduce to ones inspherical trigonometry. For a sphere the solutions to these problems are simple exercises in spherical trigonometry, whose solution is given by formulas for solving a spherical triangle.

Key words: *Geodetic problems, ellipsoid, sphere*

INTRODUCTION

Based on observations made on the terrestrial surface, projected on the reference ellipsoid, we are going to establish by calculation the positions of the first order geodetic points in a particular coordinate system used on the ellipsoid. The ultimate goal of the calculations performed on the reference ellipsoid is the determination of geodetic coordinates, latitude B and longitude L of the points from geodetic networks support which are obtained through a series of rigorous processing operations of astronomo-geodesic determinations following several phases: calculation of coordinates in the preliminary stage necessary interim processing and calculation of final coordinates after finishing proper compensation Therefore, it can be appreciated that this sort of calculation occupy a very important volume, which is why they are known in the literature as well as the solving of basic geodetic problems.The basic first geodesic problem, called direct geodetic problem, consists of determining the geodetic coordinates B2, L2 of point S2 (Figure 1) and A2 geodetic azimuth (also called inverse geodetic azimuth) depending on the coordinates B1, L1 of the S1 point, geodetic azimuth A1 (also called direct geodetic azimuth) and the length of the geodetic line s, between points S1 and S2 (Moldoveanu, 2002).

The successive use of direct geodetic problem is also known as coordinated transportation (Ortelecan, 2006).

The second basic geodetic problem, also called inverse geodetic problem consists in determining the length of the geodetic line s and direct geodetic azimuth A1 and inverse geodetic azimuth A2, when the geodetic coordinates of points S1 and S2 are known.Geodetic inverse problem is used to deduct the original elements (geodetic azimuth and distance) necessary to determine the coordinates of all points of a geodetic network, as well as a method to check the calculations made at direct geodetic problem (Ortelecan and Sălăgean, 2014).

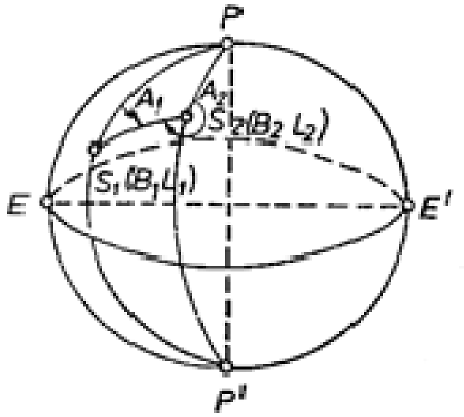

Figure 1. Sphere

MATERIALS AND METHODS

There are several criteria for the classification of methods and procedures for calculating geodetic coordinates on the reference ellipsoid, depending on the element considered mainly to be determined.
One of the current classification criteria used, considers as the geodetic main element the length of the geodetic line, denoted by s. From this point of view, can be distinguished the Taylor series method, Legendre method, in the case of small distances, below 60 km; average method arguments Gauss for medium lengths (60 < s < 600km) and in case of great distances (600< s <20000 km) using Bessel's method. Another aspect to be considered in effective solutions relates to the accuracy of the calculations of geodetic coordinates, where we can distinguish between exact methods and approximate methods.As the geodesic distances increase, accuracy in calculations has special significance.As in other geodetic calculations, but also in the geodetic based problems, it is desired that errors in the calculation to be about 10 times smaller than the average errors that characterize the operation in the field.Thus, it can be shown that in the first-order geodetic triangulation, in which intervened geodetic distances, averaging 30-40 km, it was necessary that the approximation calculation for geodetic coordinates B and L to be± 0'', 0001, for geodetic azimuths, denoted by A to be± 0'', 001, and for geodetic distances (geodetic lines), denoted by s, to be± 0, 001 m.

RESULTS AND DISCUSSIONS

The method for replacing the surface of the elipsoid with Gauss sphere

In approximation limit allowable, for short distances (s 60 km), can assimilate the ellipsoid surface with the surface of a sphere of Gauss average radius.

Solving the inverse problem

It is known :

Dl. Hoia: B=46°46'06.45315'' N
L=23°32'13.25030'' E

Dl. Steluta: B=46°48'17.54003'' N
L=23°34'56.24439'' E

We will refer to figures 2 a and 2 b, on which we , as a first step , write formulas for the half the sum and half the difference for sinus and cosines in spherical triangles:

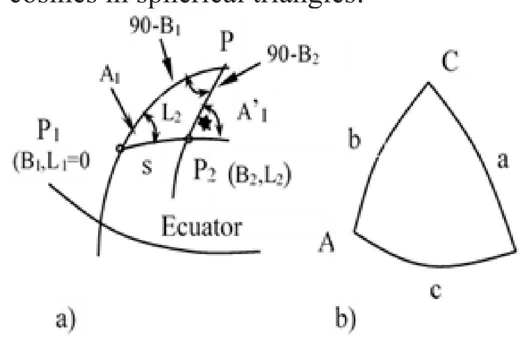

Figure 2.a.The triangle on the sphere;

2b. Corresponding spherical triangle

$$sin\frac{a}{2}cos\frac{B-C}{2} = sin\frac{b+c}{2}sin\frac{A}{2}$$
$$sin\frac{a}{2}sin\frac{B-C}{2} = sin\frac{b-c}{2}cos\frac{A}{2} \qquad (1.1)$$

$$\cos\frac{a}{2}\cos\frac{B+C}{2} = \cos\frac{b+c}{2}\sin\frac{A}{2}$$

$$cos\frac{a}{2}sin\frac{B+C}{2} = cos\frac{b-c}{2}cos\frac{A}{2}$$

By identifying P1PP2 and CAB triangles, result:

A = L2 a = s

B = 180- $A_1^{'}$ b = 90 – B1(1.2)

C = A1 c = 90 – B2

and with their help is calculated:

$$\frac{B-C}{2}=\frac{180^0-A_1^{'}-A_1}{2}=90^0-\frac{A_1^{'}+A_1}{2};$$

$$\frac{b-c}{2}=\frac{90^0-B_1-(90^0-B_2)}{2}=\frac{B_2-B_1}{2} \quad (1.3)$$

$$\frac{B+C}{2}=\frac{180^0-A_1^{'}+A_1}{2}=90^0-\frac{A_1^{'}-A_1}{2};$$

$$\frac{b+c}{2}=\frac{180^0-B_1-B_2}{2}=90^0-\frac{B_1+B_2}{2}$$

$$\frac{A}{2}=\frac{L_2}{2};\quad \frac{a}{2}=\frac{s}{2}$$

By replacing the equality (1.3 .) to (1.1.) there is obtained:

$$\text{I}\quad \sin\frac{s}{2}\sin\frac{A_1^{'}+A_1}{2}=\cos\frac{B_2+B_1}{2}\sin\frac{L_2}{2}$$

$$\text{II}\quad \sin\frac{s}{2}\cos\frac{A_1^{'}+A_1}{2}=\sin\frac{B_2-B_1}{2}\cos\frac{L_2}{2} \quad (1.4)$$

$$\text{III}\quad \cos\frac{s}{2}\sin\frac{A_1^{'}-A_1}{2}=\sin\frac{B_2+B_1}{2}\sin\frac{L_2}{2}$$

$$\text{IV}\quad \cos\frac{s}{2}\cos\frac{A_1^{;}-A_1}{2}=\cos\frac{B_2-B_1}{2}\cos\frac{L_2}{2}$$

Dividing equalities I to II and III to IV we obtain:

$$tg\frac{A_1^{i}+A_1}{2}=\frac{\cos\frac{B_2+B_1}{2}}{\sin\frac{B_2-B_1}{2}}tg\frac{L_2}{2}$$

=0,851380316

$$tg\frac{A_1^{'}-A_1}{2}=\frac{\sin\frac{B_2+B_1}{2}}{\cos\frac{B_2-B}{2}}tg\frac{L_2}{2}$$

=0,000287959

In the relations (1.5) , the coordinates of " P1 " and " P2 " being known, result that“B1”, “B2” and“L2”are known;so by solving the system of equations can be determined azimuths“A1” and „$A_1^{'}$”.

$A_1^{'}$+A1 =2arctg0,851380316 =80,82083795

$A_1^{'}$-A1=2arctg 0,000287959 = 0,032997671 (1.6)

By solving the systemwe get:

* $A_1^{'}$=(80,82083795+0,032997671)/2 =40°,42691781 =40°25’36’’,90411564

*A1=(80,82083795-0,032997671)/2 =40°,39392014 = 40°23’38’’,11249892

To calculate the value of the geodetic line, equalities I are devided to equalities III, and for verification, equalities II are devided to equalities IV:

$$tg\frac{s}{2}=\frac{\sin\frac{A_1^{'}-A_1}{2}}{\sin\frac{A_1^{'}+A_1}{2}}ctg\frac{B_2+B_1}{2}$$

=0,00041733

s’ = 2 arctg 0,00041733 =0,047822488

s=s’R=5317,716276

$$tg\frac{s}{2}=\frac{\cos\frac{A_1^{'}-A_1}{2}}{\cos\frac{A_1^{'}+A_1}{2}}tg\frac{B_2-B_1}{2}$$

=0,00041733

s’ = 2 arctg 0,00041733 =0,047822488

s=s’R=5317,716276 m

Solving the direct problem

We consider the points P1 and P2 (Fig .3)located on a sphere of average radius, calculated based on the center of gravity of the spherical triangle P1PP2 .

Geodetic coordinates of the point P1 (Dl Hoia) are known,

B=46°46’06’’.45315N L=23°34’56’’.24439 E

Azimuth direction of P1P2:

A1 =40,25,36,90411564

length of the geodesic line

s =5317,716276 m.

We will determine the coordinates of the point P2 (B2 , L2) and inverse azimuth denoted by A2.

To facilitate relations writing we choose a certain spherical triangle whose elements are represented in Fig .3b .

To solve the problem, we write formulas of three adjacent elements in this triangle and we get:

$$\sin\frac{a}{2}\cdot\cos\frac{B-C}{2}=\sin\frac{b+c}{2}\cdot\sin\frac{A}{2};$$
$$\sin\frac{a}{2}\cdot\sin\frac{B-C}{2}=\sin\frac{b-c}{2}\cdot\cos\frac{A}{2};$$

$$\cos\frac{a}{2}\cdot\cos\frac{B+C}{2}=\cos\frac{b+c}{2}\cdot\sin\frac{A}{2};$$
$$\sin\frac{a}{2}\cdot\sin\frac{B+C}{2}=\cos\frac{b-c}{2}\cdot\cos\frac{A}{2}; \quad (2.1)$$

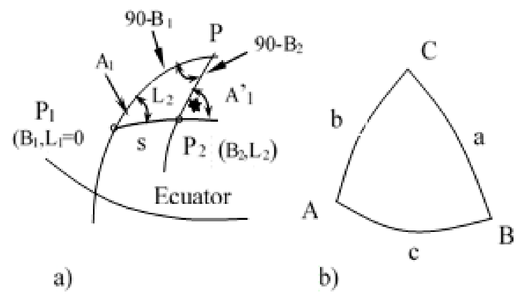

Figure 3.a.The triangle on the sphere;

3b. Corresponding spherical triangle

Spherical triangle ABC identified with spherical triangleP1P2P leads to the equalities:

A = A1 a = 900 – B2

B = 1800-A'1 b = 900 – B1 (2.2)

C = L2 c = s'

and with their help:

$$\frac{B-C}{2}=\frac{180^0-A_1^{'}-L_2}{2}=90^0-\frac{A_1^{'}+L_2}{2};\quad \frac{b+c}{2}=\frac{90^0-B_1+s}{2}$$
$$\frac{B+C}{2}=\frac{180^0-A_1^{'}+L_2}{2}=90^0-\frac{A_1^{'}-L_2}{2};\quad \frac{b-c}{2}=\frac{90^0-B_1-s}{2}$$
$$\frac{A}{2}=\frac{A_1}{2};\quad \frac{a}{2}=\frac{90^0-B_2}{2} \quad (2.3)$$

Introducing the equalities (2.3.) in (2.1.) is obtained :

$$\sin\frac{90^0-B_2}{2}\sin\frac{A_1^{'}+L_2}{2}=\sin\frac{90^0-B_1+s}{2}\sin\frac{A_1}{2} \quad \text{I} \quad \text{II}$$
$$\sin\frac{90^0-B_2}{2}\cos\frac{A_1^{'}+L_2}{2}=\sin\frac{90^0-B_1-s}{2}\cos\frac{A_1}{2} \quad \text{III}$$
$$\cos\frac{90^0-B_2}{2}\sin\frac{A_1^{'}-L_2}{2}=\cos\frac{90^0-B_1+s}{2}\sin\frac{A_1}{2} \quad \text{IV}$$
$$\cos\frac{90^0-B_2}{2}\cos\frac{A_1^{'}-B_2}{2}=\cos\frac{90^0-B_1-s}{2}\cos\frac{A_1}{2}$$

(2.4)

By dividing the relationships I and II ; III and IV is obtained :

$$tg\frac{A_1^{'}+L_2}{2}=\frac{\sin\frac{90^0-B_1+s}{2}}{\sin\frac{90^0-B_1-s}{2}}tg\frac{A_1}{2}$$

=0,368643823

$$tg\frac{A_1^{'}-L_2}{2}=\frac{\cos\frac{90^0-B_1+s}{2}}{\cos\frac{90^0-B_1-s}{2}}tg\frac{A_1}{2}$$

=0,367746477 (2.5)

Solving the system of equations (2.5) are obtained unknowns " A'1 " and " L2 " according to geodetic coordinates of the point " P1 " and geodesic length "s".

Inverse azimuth:A2=A'1+200g.

A'1+L=2arctg0,368643823 =40,4721939

A'1-L =2 arctg0,367746477 = 40,3816417

A'1=(40,4721939+40,3816417)/2=40°,42691781=40°25'36'',90411564

L2=(40,4721939-40,3816417)/2=23°,58229011=23°34'56.24439

To find the geodetic latitude of the point " P2 " (B2) relations I and III; II and IV are divided:

$$tg\frac{90-B_2}{2}=\frac{\sin\frac{A_1^{'}-L_2}{2}}{\sin\frac{A_1^{'}+L_2}{2}}tg\frac{90-B_1+s}{2}$$

=0,395878826 (2.6)

$$tg\frac{90-B_2}{2}=\frac{\cos\frac{A_1^{'}-L_2}{2}}{\cos\frac{A_1^{'}+L_2}{2}}tg\frac{90-B_1-s}{2}$$

=0,395878826

90-B2=2 arctg0,395878826 =43,19512777

B2=90-43,19512777=46°,80487223 =46°48'17.54003''

The two relations (2.6). Determine exactly the same size,the latitude " B2 " of point" P2 " .

Geodetic coordinates	Degrees [°]	Minutes [']	Seconds["]	Decimal Degrees
B1	46	46	6,45315	46,76845921
L1	23	32	13,2503	23,53701397
B2	46	48	17,54003	46,80487223
L2	23	34	56,24439	23,58229011
L=L2-L1	0	2	42,99409	0,045276136

Coordonate geodezice	Grade [°]	Minute [']	Secunde["]	Grade Decimale
B1	46	46	6,45315	46,76845921
L1	23	32	13,2503	23,53701397
A1	40	23	38,11249892	40,39392014

s[m]	s'[rad]=s/R	s'[decimal]	s'[°.'."]
5317,716276	0,0008347	0,04782249	0°2'52".16095726

CONCLUSIONS

As a result of the performed calculations it is observed that the geodetic coordinates of the second point of the direct geodetic problem are the same with the coordinates of the point of the inverse geodetic problem.

For small areas using the calculation method of average radius sphere offers acceptable accuracy for practical work.

Calculation of geodetic coordinates on the average radius sphere can also be used for large areas reported on a small scale.

In the case of large areas, when calculating geodetic coordinates on the sphere, deformations that occur when projecting the linear elements off the ellipsoid on the sphere should be taken into account.

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