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COMPENSATION OF A TRILATERATION NETWORK AND VERIFICATION OF ERRORS RANDOMNESS

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Abstract

This paper presents a method of determining the coordinates of new points based on the measured distances (trilateration) using the indirect measurement method. This method is treated theoretically and numerically using Gauss-Markov method, the matrix treating. Another contribution consists of the Young Test to verify the random errors.

Key words*: compensation, measurements, network, trilateration, Young Test.*

INTRODUCTION

Planimetric support networks are formed of points, which joined together with imaginary lines form a series of adjacent triangles. The trilateration participates in creating the geodetic network, all the points located on the surface of the Earth, for which the coordinates are known in a reference system. The state geodetic network, created separately by triangulation and levelling, is the main support network for all topo-geodetic and photogrammetric work. It is divided in orders: I, II, III and IV. The state triangulation network was completed with a thickening network of order V. (Moldoveanu, 2000)

There were defined several classification criteria for networks, but by the type of network measurements exists:

- triangulation networks;
- trilateration networks;
- networks formed with global positioning stations;
- mixed networks.

Trilateration is the process of measuring distances (edges) in planimetric support networks in order to determine the coordinates of the points that form these networks.

As electronic distance measuring equipment provides great accuracy and as linear measurement is much easier than the angular measurement, trilateration can be considered as one of the most economic methods to create, rehabilitate and thicken the planimetric support networks.

To execute a trilateration every point of the network has to be accessible because at each measured edge on one end will be installed the instrument and on the other the reflector. It is generally stationed in all the points and the edges are measured in both directions. (Popia, 2005)

MATERIALS AND METHODS

On a set of distance measurements effectuated with the indirect method in a network formed of 2 points of known rectangular coordinates (X, Y) and 5 new points, the coordinates for the new points will be determined (Figure 1).

The distances were measured in both directions in order to benefit of a rigorous compensation.

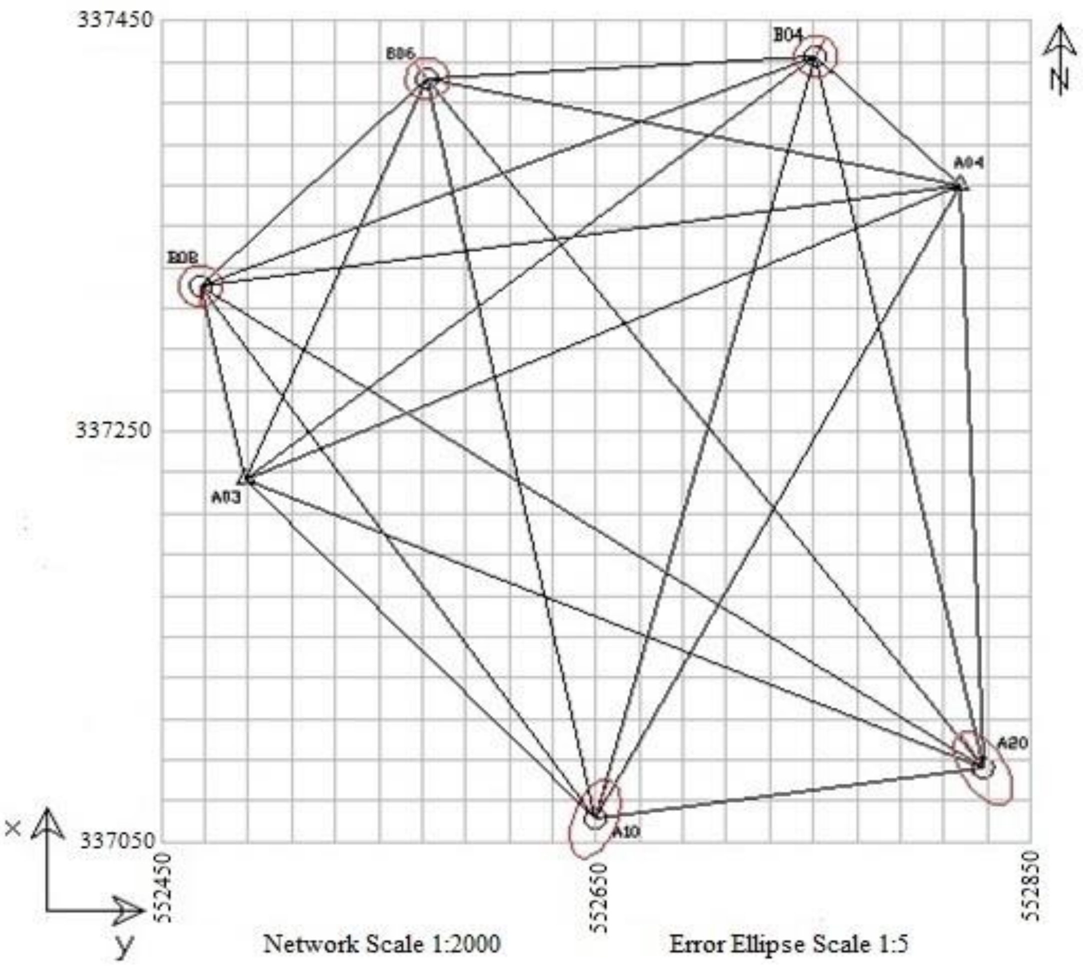

Figure 1. The trilateration network

In order to compensate the network the Gauss-Markov method is applied, which involves the matrix treating.

RESULTS AND DISCUSSIONS

Compensating a trilateration network involves going through based stages, beginning by writing the correction equations and calculating the weights.
The weights can be calculated with the relation:

$p_i = \frac{1}{(s'_D)^2}$ sau $p_i = \frac{D_{min}}{D_i}$, where s'_D is the average error of the series of observations made on that edge and D_{min} is the length of the smallest edge measured in the network, which receives the value 1 as its weight.

By adding adjustments to the provisional values (Table 1) there will be determined the most probable values of the parameters. (Moldoveanu, 2000)

Table 1. Provisional coordinates of the new points points

Point	X° [m]	Y° [m]	Θ° [g c cc]	Point	X° [m]	Y° [m]
A03	337226.600	552488.783	386.1486	B08	337320.884	552467.939
A04	337370.105	552817.167	291.0860			
B08	337320.884	552467.939				
		552467.939				
A03	337226.600	552488.783	25.7477	B06	337421.866	552572.365
A04	337370.105	552817.167	313.2653			
B06	337421.866	552572.365				
		552572.365				
A03	337226.600	552488.783	57.5788	B04	337432.742	552750.940
A04	337370.105	552817.167	348.2269			
B04	337432.742	552750.940				
		552750.940				
A03	337226.600	552488.783	124.9872	A20	337086.163	552828.021
A04	337370.105	552817.167	197.5676			
A20	337086.163	552828.021				
		552828.021				
A03	337226.600	552488.783	150.8739	A10	337061.307	552649.599
A04	337370.105	552817.167	231.6514			
A10	337061.307	552649.599				
		552649.599				

The corrections are called coordinates increases and are denoted dX, respectively Dy.

$X_i = X^o_i + dX_i$ $X_j = X^o_j + dX_j$

$Y_i = Y^o_i + dY_i$ $Y_j = Y^o_j + dY_j$

This will get:

$$v_{ij}^{D} = \cos\theta_{ij}^{0} dX_{j} + \sin\theta_{ij}^{0} dY - \cos\theta_{ij}^{0} dX_{i}$$
$$- \sin\theta_{ij}^{0} dY_{i} + (D_{ij}^{0} - D_{ij}^{*})$$

Where θ_{ij}^{0} and D_{ij}^{0} are calculated with the provisional coordinates of the points and D_{ij}^{*} is the measured distance.

The correction equation for the measured distance between two new points “i” and “j” is calculated using the formula:

$$v_{ij}^{D} = A_{ij} dX_{j} + Bij dY - Aij dX_{i} - Bij dY_{i} + l_{ij}^{D}$$

where $l_{ij}^{D} = D_{ij}^{0} - D_{ij}^{*}$.

The form for the correction equation for the measured distance between an old point “i” and a new point “j”:

$$v_{ij}^{D} = A_{ij} dX_{j} + Bij dY_{j} + l_{ij}^{D}$$

The form for the correction equation for the measured distance between a new point “i” and an old point “j”:

$$v_{ij}^{D} = -Aij dX_{i} - Bij dY_{i} + l_{ij}^{D}$$

Between two old points distance measurements are not performed.

To compensate the network it’s necesary to solve the normal system of equations. Based upon the calculated coefficients for the unknown elements of the linear system of corrections will be issued the coefficients matrix, the matrix A. Starting from the general form, the matrix form, of the corection equations: $V = Ax + l$. The formed system is an indeterminate compatible system, with the following notations:

V – the measurement corrections vector;

A – the coeffcients of the correction equations matrix;

x – the coordinate increases vector (unknowns vector)

l – the free terms vector .

Applying the least square method $V^{T}pV \rightarrow \min$ there will be determined formulas for the coordinate increases vector and for the corrections vector. Using the N matrix, the normal matrix, one can determine the unknowns vector.

$$N = A^{T}PA$$
$$A^{T}PAx + A^{T}Pl = 0$$
$$x = -N^{-1}A^{T}Pl$$
$$V = Ax + l$$

The normal system is compatible determined, so the values of the unknowns can be uniquely determined. (Moldoveanu, 2000) The results of the matrix calculus is presented below:

A =

0.9487	-0.2097	0	0	0	0	0	0	0	0	0.0001
0	0	0.6023	0.2578	0	0	0	0	0	0	0.0002
0	0	0	0	0.3232	0.4110	0	0	0	0	0.0003
0	0	0	0	0	0	-0.1906	0.4604	0	0	0.0004
0	0	0	0	0	0	0	0	-0.4506	0.4384	0.0002
0.9487	-0.2097	0	0	0	0	0	0	0	0	0.0001
-0.5507	-0.5694	0.5507	0.5694	0	0	0	0	0	0	0.0001
-0.2012	-0.5090	0	0	0.2012	0.5090	0	0	0	0	0.0003
-0.0710	-0.5034	0	0	0	0	0	0	0	0	0.0004
0.2515	-0.3858	0	0	0	0	-0.2515	0.3858	0	0	0.0004
0.4395	-0.3076	0	0	0	0	0	0	-0.4395	0.3076	0.0003
0	0	0.6023	0.2578	0	0	0	0	0	0	0.0002
-0.5507	-0.5695	0.5507	0.5695	0	0	0	0	0	0	0.0001
0	0	-0.0434	-0.7125	0.0434	0.7125	0	0	0	0	0.0002
0	0	0.1249	-0.5905	0	0	0	0	0	0	0.0003
0	0	0.3698	-0.2816	0	0	-0.3698	0.2816	0	0	0.0004
0	0	0.4862	-0.1041	0	0	0	0	-0.4862	0.1041	0.0004
0	0	0	0	0.3232	0.4110	0	0	0	0	0.0003
-0.2012	-0.5090	0	0	0.2012	0.5090	0	0	0	0	0.0003
0	0	-0.0434	-0.7125	0.0434	0.7125	0	0	0	0	0.0002
0	0	0	0	0.6871	-0.7265	0	0	0	0	0.0001
0	0	0	0	0.4946	-0.1100	-0.4946	0.1100	0	0	0.0004
0	0	0	0	0.4694	0.1281	0	0	-0.4694	-0.1281	0.0004
0.1052	-0.4974	0	0	0	0	0	0	0	0	0.0004
0	0	0.1249	-0.5905	0	0	0	0	0	0	0.0003
0	0	0	0	0.6871	-0.7265	0	0	0	0	0.0001
0	0	0	0	0	0	-0.5660	0.0216	0	0	0.0003
0	0	0	0	0	0	0	0	-0.4477	-0.2429	0.0004
0	0	0	0	0	0	-0.1906	0.4604	0	0	0.0004
0.2515	-0.3858	0	0	0	0	-0.2515	0.3858	0	0	0.0004
0	0	0.3698	-0.2816	0	0	-0.3698	0.2816	0	0	0.0004
0	0	0	0	0.4946	-0.1100	-0.4946	0.1100	0	0	0.0004
0	0	0	0	0	0	-0.5660	0.0216	0	0	0.0003
0	0	0	0	0	0	0.0982	0.7045	-0.0982	-0.7045	0.0002
0	0	0	0	0	0	0	0	-0.4506	0.4384	0.0002
0.4395	-0.3075	0	0	0	0	0	0	-0.4395	0.3075	0.0003
0	0	0.4862	-0.1041	0	0	0	0	-0.4862	0.1041	0.0004
0	0	0	0	0.4694	0.1281	0	0	-0.4694	-0.1281	0.0004
0	0	0	0	0	0	0	0	-0.4477	-0.2429	0.0004
0	0	0	0	0	0	0.0982	0.7046	-0.0982	-0.7046	0.0002

l=

-0.0018
0.0001
-0.0028
-0.0009
-0.0019
-0.0009
-0.0002
0.0002
0.0008
0.0000
0.0010
0.0001
0.0014
-0.0001
0.0004
0.0003
0.0001
-0.0001
0.0007
-0.0001
0.0004
0.0013
-0.0001
-0.0002
-0.0009
-0.0036
0.0002
-0.0003
0.0001
0.0000
-0.0011
0.0003
0.0030
0.0000
0.0000
-0.0001
-0.0014
-0.0021
0.0017
0.0035

V =

-0.001
0.001
-0.001
-0.001
-0.001
0.000
-0.001
0.000
0.001
0.000
0.002
0.001
0.000
0.000
0.001
0.000
0.001
0.001
0.000
0.000
0.002
0.001
0.000
0.000
0.000
-0.002
-0.001
-0.001
0.000
0.000
-0.001
0.000
0.002
-0.001
0.001
0.000
-0.001
-0.002
0.001
0.002

N^{-1} =

0.434	0.005	0.216	0.073	0.140	0.135	-0.081	0.008	-0.007	-0.075	-212.429
0.005	1.259	-0.291	0.584	-0.340	-0.007	0.763	-0.315	0.804	0.195	1520.715
0.216	-0.291	0.960	-0.298	0.448	0.228	-0.415	0.361	-0.398	-0.015	-1128.255
0.073	0.584	-0.298	0.773	-0.175	0.135	0.354	-0.176	0.429	0.093	843.283
0.140	-0.340	0.448	-0.175	0.997	0.355	-0.347	0.594	-0.439	0.192	-1235.153
0.135	-0.007	0.228	0.135	0.355	0.605	-0.287	0.323	-0.250	0.105	-628.812
-0.081	0.763	-0.415	0.354	-0.347	-0.287	1.794	-0.200	1.110	0.362	1866.808
0.008	-0.315	0.361	-0.176	0.594	0.323	-0.200	1.587	-0.522	0.752	-1363.538
-0.007	0.804	-0.398	0.429	-0.439	-0.250	1.110	-0.522	1.605	0.196	1979.238
-0.075	0.195	-0.015	0.093	0.192	0.105	0.362	0.752	0.196	1.116	-10.828
-212.429	1520.715	-1128.255	843.283	-1235.153	-628.812	1866.808	-1363.538	1979.238	-10.828	3916660.294

x =

0.001
0.002
0.001
0.000
0.002
0.000
0.004
0.000
0.002
0.003
2.076

The compensated values are determined by adding the systems solutions to the provisional values (Table 2).
After determining the compensated coordinates, the compensation of the network can be finished (Table 3).

Table 2. The compensated coordinates

Point	X° [m]	Y° [m]	dx [m]	dy [m]	X [m]	Y [m]
B08	337320.884	552467.939	0.001	0.002	337320.885	552467.941
B06	337421.866	552572.365	0.001	0.000	337421.866	552572.365
B04	337432.742	552750.940	0.002	0.000	337432.744	552750.940
A20	337086.163	552828.021	0.004	0.000	337086.167	552828.022
A10	337061.307	552649.599	0.002	0.003	337061.309	552649.602

Any processing of observations in a geodetic network ends with the calculus of precision assessment indicators.
The standard deviation of the unit weight:

$$s_0 = \sqrt{\frac{V^T \cdot P \cdot V}{m-n}}$$

where m is the number of measurements and n is the number of unknowns.
The standard deviation of a compensated measurement:

$$s_{mij} = \frac{s_0}{\sqrt{p_i}}$$

The standard deviation of the unknowns:

$$s_{x_i} = s_0 \cdot \sqrt{q_{x_i x_i}}$$

$$s_{y_i} = s_0 \cdot \sqrt{q_{y_i y_i}}$$

The standard deviation to determine the position of the point:

$$s_{p_i} = \sqrt{s_{x_i}^2 + s_{y_i}^2}$$

The standard deviation on the network:

$$s_t = \frac{\sum s_{p_i}}{n}$$

Where n is the number of new points. (Voineagu, 2007)
The values obtained for the standard deviations are:

$s_0 = 0.001$ m

$s_{xB08} = 0.001$ m $\quad s_{yB08} = 0.002$ m

$s_{xB06} = 0.001$ m $\quad s_{yB06} = 0.001$ m

$s_{xB04} = 0.001$ m $\quad s_{yB04} = 0.001$ m

$s_{xA20} = 0.002$ m $\quad s_{yA20} = 0.002$ m

$s_{xA10} = 0.002$ m $\quad s_{yA10} = 0.001$ m

$s_{pB08} = 0.002$ m

$s_{pB06} = 0.002$ m

$s_{pB04} = 0.002$ m

$s_{pA20} = 0.002$ m

$s_{pA10} = 0.002$ m

$s_t = 0.002$ m

The planimetric point position depends on two parameters, X and Y. The confidence domain of the planimetric position of a point is given by the invariant called error ellipse (Figure 2).

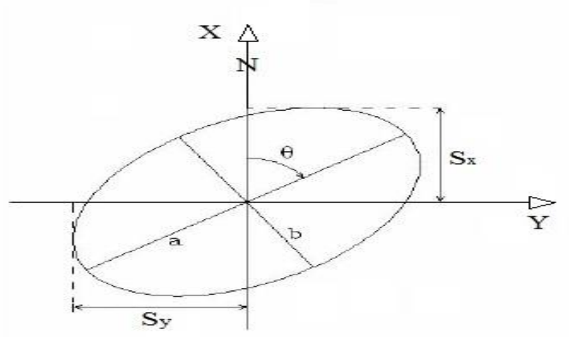

Figure 2. Error ellipse

After compensating the point P_j, the coordinates (X_j, Y_j) were obtained and the two-dimensional block:

$$Q_{jj} = \begin{pmatrix} q_{x_j x_j} & q_{x_j y_j} \\ q_{y_j x_j} & q_{y_j y_j} \end{pmatrix}$$

This block is extracted from the general matrix of cofactors: $Q_{xx} = N^{-1}$.
The error ellipse elements (Table 4) are:

- the semi-major axis: $a = S_0 \sqrt{\lambda_1}$
- the semi-minor axis: $b = S_0 \sqrt{\lambda_2}$
- the angle of orientation (the orientation of the semi-major axis to the axis X):

$$\Theta = \frac{1}{2} \operatorname{arctg} \frac{2q_{xy}}{q_{xx} - q_{yy}}$$

where: $\lambda_{1,2} = \frac{q_{xx} + q_{yy}}{2} \pm \frac{1}{2}\sqrt{(q_{xx} - q_{yy})^2 + 4q_{xy}^2}$.

Table 4. The error ellipse elements

Point	a [m]	b [m]	Θ [g]
B08	0.001	0.001	199.6017
B06	0.001	0.001	359.7055
B04	0.001	0.001	33.9736
A20	0.002	0.001	365.2458
A10	0.002	0.001	21.5545

Table 3. The compensation of the network

PS	PV	Θ° [g c cc]	D* [m]	D° [m]	Unknowns										l_{ij} [m]	P_{ij}	dD [m]	v [m]	Control	
					dx_{B08} [m]	dy_{B08} [m]	dx_{B06} [m]	dy_{B06} [m]	dx_{B04} [m]	dy_{B04} [m]	dx_{A20} [m]	dx_{A20} [m]	dx_{A10} [m]	dy_{A10} [m]					$D_{corected}$ [m]	$D_{calculated}$ [m]
					0.001	0.002	0.001	0.000	0.002	0.000	0.004	0.000	0.002	0.003						
A03	B08	386.1486	96.562	96.560	0.9764	-0.2159	0	0	0	0	0	0	0	0	-0.002	0.944	0.001	-0.001	96.561	96.561
	B06	25.7477	212.402	212.402	0	0	0.9193	0.3935	0	0	0	0	0	0	0.000	0.429	0.001	0.000	212.403	212.403
	B04	57.5788	333.503	333.498	0	0	0	0	0.6181	0.7861	0	0	0	0	-0.005	0.273	0.001	-0.005	333.499	333.499
	A04	73.7717	358.371	358.371	0	0	0	0	0	0	0	0	0	0	0.000	0.254	0.000	-0.001	358.371	358.371
	A20	124.9872	367.160	367.158	0	0	0	0	0	0	-0.3825	0.9240	0	0	-0.002	0.248	-0.001	-0.004	367.157	367.157
	A10	150.8739	230.619	230.616	0	0	0	0	0	0	0	0	-0.7167	0.6973	-0.003	0.395	0.001	-0.003	230.617	230.617
B08	A03	186.1486	96.561	96.560	0.9764	-0.2159	0	0	0	0	0	0	0	0	-0.001	0.944	0.001	0.000	96.561	96.561
	B06	51.0671	145.266	145.266	-0.6952	-0.7189	0.6952	0.7189	0	0	0	0	0	0	0.000	0.628	-0.002	-0.002	145.264	145.264
	B04	76.0368	304.305	304.305	-0.3676	-0.9300	0	0	0.3676	0.9300	0	0	0	0	0.000	0.300	-0.002	-0.002	304.304	304.304
	A04	91.0860	352.678	352.680	-0.1396	-0.9902	0	0	0	0	0	0	0	0	0.002	0.258	-0.002	-0.001	352.678	352.678
	A20	136.7760	429.829	429.829	0.5461	-0.8377	0	0	0	0	-0.5461	0.8377	0	0	0.000	0.212	-0.003	-0.004	429.826	429.826
	A10	161.1271	316.827	316.829	0.8193	-0.5734	0	0	0	0	0	0	-0.8193	0.5734	0.002	0.288	0.000	0.001	316.829	316.829
B06	A03	225.7477	212.402	212.402	0	0	0.9193	0.3935	0	0	0	0	0	0	0.000	0.429	0.001	0.000	212.403	212.403
	B08	251.0671	145.264	145.266	-0.6952	-0.7189	0.6952	0.7189	0	0	0	0	0	0	0.002	0.628	-0.002	0.000	145.264	145.264
	B04	96.1273	178.906	178.906	0	0	-0.0608	-0.9982	0.0608	0.9982	0	0	0	0	0.000	0.510	0.000	-0.001	178.906	178.906
	A04	113.2653	250.214	250.215	0	0	0.2069	-0.9784	0	0	0	0	0	0	0.001	0.364	0.000	0.000	250.215	250.215
	A20	158.5652	421.967	421.968	0	0	0.7956	-0.6059	0	0	-0.7956	0.6059	0	0	0.001	0.216	-0.002	-0.003	421.965	421.965
	A10	186.5661	368.738	368.738	0	0	0.9778	-0.2095	0	0	0	0	-0.9778	0.2095	0.000	0.247	-0.001	-0.001	368.738	368.738
B04	A03	257.5788	333.498	333.498	0	0	0	0	0.6181	0.7861	0	0	0	0	0.000	0.273	0.001	0.000	333.499	333.499
	B08	276.0368	304.304	304.305	-0.3676	-0.9300	0	0	0.3676	0.9300	0	0	0	0	0.001	0.300	-0.002	-0.001	304.304	304.304
	B06	296.1273	178.906	178.906	0	0	-0.0608	-0.9982	0.0608	0.9982	0	0	0	0	0.000	0.510	0.000	-0.001	178.906	178.906
	A04	148.2269	91.156	91.156	0	0	0	0	0.6871	-0.7265	0	0	0	0	0.000	1.000	0.001	0.002	91.158	91.158
	A20	186.0679	355.045	355.048	0	0	0	0	0.9761	-0.2171	-0.9761	0.2171	0	0	0.003	0.257	-0.002	0.000	355.046	355.046
	A10	216.9565	385.012	385.012	0	0	0	0	0.9647	0.2632	0	0	-0.9647	-0.2632	0.000	0.237	-0.001	-0.002	385.011	385.011
A04	A03	273.7717	358.371	358.371	0	0	0	0	0	0	0	0	0	0	0.000	0.254	0.000	-0.001	358.371	358.371
	B08	291.0860	352.680	352.680	0.2069	-0.9784	0	0	0	0	0	0	0	0	0.000	0.258	-0.002	-0.003	352.678	352.678
	B06	313.2653	250.216	250.215	0	0	0.2069	-0.9784	0	0	0	0	0	0	-0.001	0.364	0.000	-0.002	250.215	250.215
	B04	348.2269	91.160	91.156	0	0	0	0	0.6871	-0.7265	0	0	0	0	-0.004	1.000	0.001	-0.002	91.158	91.158
	A20	197.5676	284.149	284.149	0	0	0	0	0	0	-0.9993	0.0382	0	0	0.000	0.321	-0.004	-0.004	284.146	284.146
	A10	231.6514	351.334	351.333	0	0	0	0	0	0	0	0	-0.8789	-0.4769	-0.001	0.259	-0.003	-0.004	351.330	351.330
A20	A03	324.9872	367.158	367.158	0	0	0	0	0	0	-0.3825	0.9240	0	0	0.000	0.248	-0.001	-0.002	367.157	367.157
	B08	336.7760	429.829	429.829	0.5461	-0.8377	0	0	0	0	-0.5461	0.8377	0	0	0.000	0.212	-0.003	-0.004	429.826	429.826
	B06	358.5652	421.970	421.968	0	0	0.7956	-0.6059	0	0	-0.7956	0.6059	0	0	-0.002	0.216	-0.002	-0.006	421.965	421.965
	B04	386.0679	355.047	355.048	0	0	0	0	0.9761	-0.2171	-0.9761	0.2171	0	0	0.001	0.257	-0.002	-0.002	355.046	355.046
	A04	397.5676	284.144	284.149	0	0	0	0	0	0	-0.9993	0.0382	0	0	0.005	0.321	-0.004	0.001	284.146	284.146
	A10	291.1879	180.145	180.145	0	0	0	0	0	0	0.1380	0.9904	-0.1380	-0.9904	0.000	0.506	-0.002	-0.003	180.143	180.143
A10	A03	350.8739	230.616	230.616	0	0	0	0	0	0	0	0	-0.7167	0.6973	0.000	0.395	0.001	0.000	230.617	230.617
	B08	361.1271	316.829	316.829	0.8193	-0.5734	0	0	0	0	0	0	-0.8193	0.5734	0.000	0.288	0.000	-0.001	316.829	316.829
	B06	386.5661	368.741	368.738	0	0	0.9778	-0.2095	0	0	0	0	-0.9778	0.2095	-0.003	0.247	-0.001	-0.004	368.738	368.738
	B04	16.9565	385.016	385.012	0	0	0	0	0.9647	0.2632	0	0	-0.9647	-0.2632	-0.004	0.237	-0.001	-0.006	385.011	385.011
	A04	31.6514	351.330	351.333	0	0	0	0	0	0	0	0	-0.8789	-0.4769	0.003	0.259	-0.003	0.000	351.330	351.330
	A20	91.1879	180.140	180.145	0	0	0	0	0	0	0.1380	0.9904	-0.1380	-0.9904	0.005	0.506	-0.002	0.002	180.143	180.143

The error ellipse is used in determining the confidence domain of the planimetric position of the points coordinates, determining the directions after which the error has extremely high or low values, determining the error in any direction, optimizing the geodetic network. (Nistor, 1998)

The compensation of geodetic measurements and the statistical analysis of the results is based on the randomness of the measurement errors. R. L. Young (1941) suggested the next statistics (Table 5), used to detect the non-random feature:

$$\delta^2 = \frac{1}{n-1}\sum_{i=1}^{n-1}(x_{i+1} - x_i)^2$$

The statistics δ^2 is called the square average of successive differences.

The next statistics will be used to test the non-random feature:

$\theta = \frac{\delta^2}{S^2}$ (Von Neuman, 1941)

The statistics compares two estimators of the theoretic dispersion in the distribution $N(\mu, \sigma^2)$.

The critical values of the statistics (θ_{critic}) were tabled by Hart (1942). In that table are calculated lower critical values ($\theta_{c.i.} = \theta_{n,a}$) and upper critical values ($\theta_{c.s.} = \theta_{n,a}$) for the risk coefficient $\alpha = 0.05$ and $\alpha = 0.01$.

The decision to accept a null hypothesis, that the selection has a non-random feature, is taken if: $\theta_{c.i.} \le \theta_{calc.} \le \theta_{c.s.}$

If the selection volume is n > 25, then the statistics $\theta' = 1 - \frac{\theta}{2}$ is normally distributed $N\left(0, \frac{n-2}{n^2-1}\right)$. In this case the statistics is calculated with the formula: $\theta' = \theta_{calc.} = \frac{\delta^2}{2S^2}$.

It is compared with the critical value:

$$\theta_{critic} = \theta_{n,\alpha} = 1 - k_\alpha \sqrt{\frac{n-2}{n^2-1}}$$

If $\theta_{calc.} \ge \theta_{critic}$, then the hypothesis of a random feature is rejected. Otherwise it is accepted the alternative hypothesis that the values have a random feature. (Laurenzi, 2010)

The values that determine the random feature are:

$v_M = -0.0016$ m $\theta = 1.8324$ g

$S^2 = 0.000001$ m^2 $\theta' = 0.0838$ g

$\delta^2 = 0.000005$ m^2 $\theta_{critic} = 0.7464$

Table 5. The Young Test

No.	v [m]	v_M - v [m]	$(v_M - v)^2$ [m^2]	$v_{i+1} - v_i$ [m]	$(v_{i+1} - v_i)^2$ [m^2]
1	-0.0014	-0.0002	0.000000	0.0017	0.0000028
2	0.0002	-0.0019	0.000004	-0.0051	0.0000257
3	-0.0048	0.0032	0.000010	0.0012	0.0000014
4	-0.0037	0.0020	0.000004	0.0008	0.0000006
5	-0.0029	0.0013	0.000002	0.0025	0.0000062
6	-0.0004	-0.0012	0.000001	-0.0017	0.0000030
7	-0.0022	0.0005	0.000000	0.0003	0.0000001
8	-0.0019	0.0003	0.000000	0.0007	0.0000005
9	-0.0012	-0.0005	0.000000	-0.0025	0.0000064
10	-0.0037	0.0021	0.000004	0.0047	0.0000223
11	0.0010	-0.0027	0.000007	-0.0008	0.0000006
12	0.0002	-0.0019	0.000004	-0.0004	0.0000002
13	-0.0002	-0.0015	0.000002	-0.0006	0.0000003
14	-0.0007	-0.0009	0.000001	0.0007	0.0000005
15	0.0000	-0.0016	0.000003	-0.0027	0.0000072
16	-0.0027	0.0010	0.000001	0.0014	0.0000021
17	-0.0012	-0.0004	0.000000	0.0014	0.0000020
18	0.0002	-0.0018	0.000003	-0.0011	0.0000011
19	-0.0009	-0.0007	0.000001	0.0002	0.0000000
20	-0.0007	-0.0009	0.000001	0.0023	0.0000053
21	0.0016	-0.0032	0.000010	-0.0014	0.0000020
22	0.0002	-0.0018	0.000003	-0.0019	0.0000038
23	-0.0018	0.0001	0.000000	-0.0010	0.0000010
24	-0.0028	0.0011	0.000001	0.0008	0.0000006
25	-0.0020	0.0004	0.000000	-0.0004	0.0000002
26	-0.0024	0.0008	0.000001	-0.0014	0.0000021
27	-0.0039	0.0022	0.000005	-0.0005	0.0000002
28	-0.0043	0.0027	0.000007	0.0027	0.0000071
29	-0.0017	0.0000	0.000000	-0.0020	0.0000041
30	-0.0037	0.0021	0.000004	-0.0020	0.0000039
31	-0.0057	0.0040	0.000016	0.0039	0.0000149
32	-0.0018	0.0002	0.000000	0.0030	0.0000088
33	0.0011	-0.0028	0.000008	-0.0038	0.0000148
34	-0.0027	0.0011	0.000001	0.0028	0.0000078
35	0.0001	-0.0017	0.000003	-0.0011	0.0000011
36	-0.0010	-0.0007	0.000000	-0.0033	0.0000106
37	-0.0042	0.0026	0.000007	-0.0015	0.0000024
38	-0.0058	0.0041	0.000017	0.0054	0.0000296
39	-0.0003	-0.0013	0.000002	0.0026	0.0000069
40	0.0023	-0.0039	0.000016		
Σ	-0.0658	0.0000	0.000000	0.0037	0.0002099

CONCLUSIONS

By checking the random nature of the experimental data there can be found their systematic errors. Knowing that only random errors carry the characteristics of random variables, the presence of systematic errors has an undesirable influence on the studied distribution.

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